

5.8 Volume

Volume: The amount of space that an object occupies, measured in cubic units.

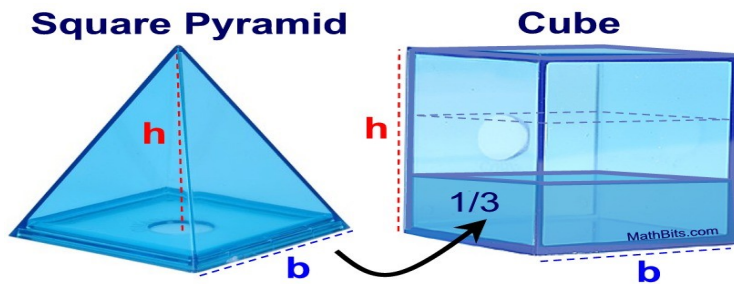


How many Pyramids fit into a prism with the same base and height?



DEMO- beads for volume

<https://www.youtube.com/watch?v=OUDjY6vJ8pw>




Summary:

The volume of a Cube with the same base and height is 3x the volume of a pyramid.

The volume of a pyramid with the same base and height is 1/3 the volume of a cube.

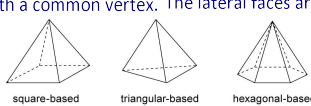
NOTE: As long as the base and the height is the same this holds true.

Prism: A solid that has two congruent and parallel faces. The lateral faces are rectangles.



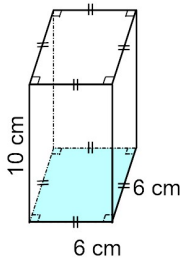
Volume = base area x height

Pyramid: A solid with a base that is a polygon and faces are triangles with a common vertex. The lateral faces are triangles.



Volume = $\frac{\text{area of base} \times \text{height}}{3}$

Ex. 1 Find the volume of the pyramid with the same base and height as the following object.



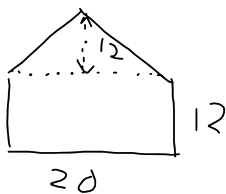
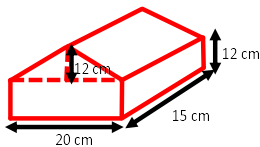
$$V_{\text{PYR}} = \frac{A_{\text{BASE}} \times h}{3}$$

$$= \frac{6 \cdot 6 \cdot 10}{3}$$

$$= \frac{360}{3}$$

$$= 120 \text{ cm}^3$$

Ex.2 Calculate volume of this solid.



$$\text{Area} = A_{\text{RECT}} + A_{\text{TRI}}$$

$$= 20(12) + \frac{20(12)}{2}$$

$$= 360 \text{ cm}^2$$

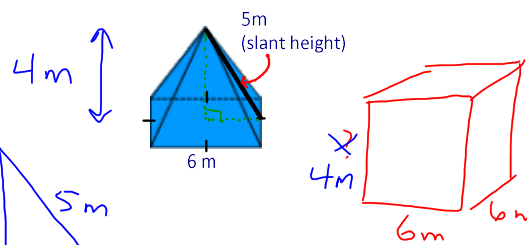
Volume

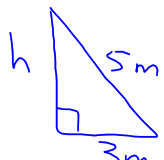
$$V = A_{\text{BASE}} \cdot \text{Height}$$

$$= 360 \cdot 15$$

$$= 5400 \text{ cm}^3$$

Ex. 3 Find the volume of the prism with the same base and height as the pyramid below.





$$h^2 + 3^2 = 5^2$$

$$h^2 + 9 = 25$$

$$h^2 = 25 - 9$$

$$h^2 = 16$$

$$h = \sqrt{16}$$

$$= 4$$

Volume

$$V = 6 \cdot 6 \cdot 4$$

$$= 144 \text{ m}^3$$

Can you guess how many cones with the same base and height fit into a cylinder?

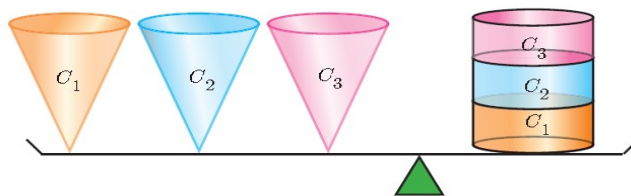
<https://www.youtube.com/watch?v=0ZACAU4SGyM>



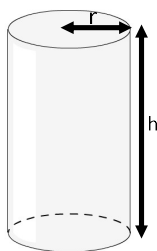
Summary:

The volume of a cylinder with the same base and height is 3x the volume of a cone.

The volume of a cone with the same base and height is 1/3 the volume of a cylinder.

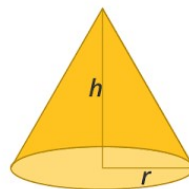


Cylinders



$$\text{Volume} = \pi r^2 h$$

Cones

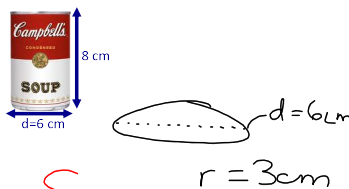


$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

Ex. 4 Calculate the capacity of the soup can.

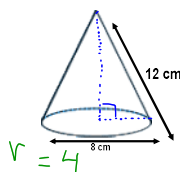
$$\begin{aligned}
 A_{\text{BASE}} &= \pi r^2 \\
 &= \pi(3)^2 \\
 &= 3.14(9) \\
 &= 28.3 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Volume} \\
 V &= A_{\text{BASE}} \cdot h \\
 &= 28.3(8) \\
 &= 84.9 \text{ cm}^3
 \end{aligned}$$



$$\begin{aligned}
 V &= \pi r^2 h \\
 &= \pi(3)^2(8)
 \end{aligned}$$

Ex. 5 a) Calculate the volume of the cone to the nearest cm^3 .



$$\begin{aligned}
 h^2 + 4^2 &= 12^2 \\
 h^2 + 16 &= 144 \\
 h^2 &= 144 - 16 \\
 &= 128 \\
 h &= \sqrt{128} \\
 &= 11.3 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 V &= \frac{1}{3} \pi r^2 h \\
 &= \frac{1}{3} \pi (4)^2 (11.3) \\
 &= 189.3 \text{ cm}^3
 \end{aligned}$$

b) What is the volume of a cylinder with the same base and height?

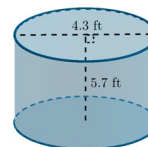
Since cone is $\frac{1}{3}$ volume of a similar cylinder

$$\begin{aligned}
 V &= 189.3 \times 3 \\
 &= 568 \text{ cm}^3
 \end{aligned}$$

Ex 6 Determine the volume of a cone that just fits inside this cylinder

$$\begin{aligned}
 V &= \frac{1}{3} \pi r^2 h \\
 h &= 5.7 \\
 r &= \frac{4.3}{2} \\
 &= 2.15
 \end{aligned}$$

$$\begin{aligned}
 V &= \frac{1}{3} \pi (2.15)^2 (5.7) \\
 &= 27.6 \text{ cm}^3
 \end{aligned}$$



Ex. 7 A cylinder has a volume of 263.9 cm^3 and a base radius of 6 cm. What is the cylinder's height?

$$\begin{aligned}
 V &= \pi r^2 h \\
 263.9 &= \pi(6)^2 h \\
 263.9 &= \pi(36) h \\
 263.9 &= 113.09 h
 \end{aligned}$$

$$\begin{aligned}
 10 &= 2x \\
 \frac{10}{2} &= x
 \end{aligned}$$

$$\frac{263.9}{113.09} = h$$

$$2.3 = h$$

\therefore The cylinder has a height of 2.3 cm