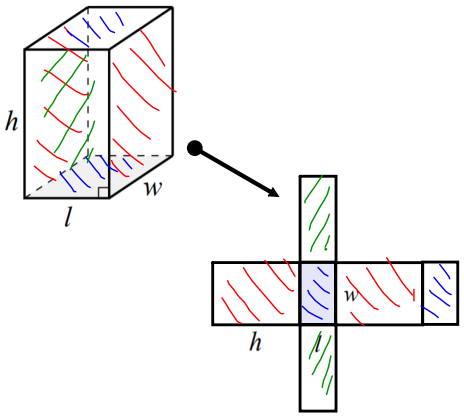
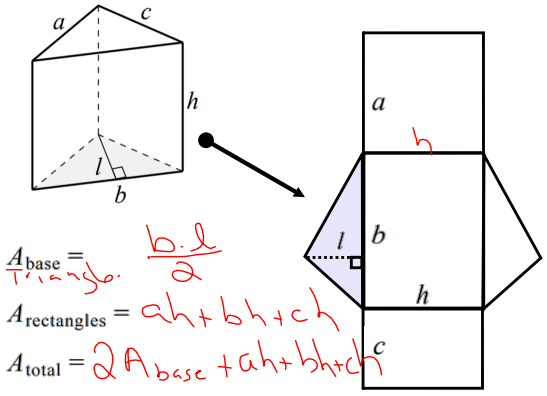
5.9 Surface Area: Prisms, Pyramids, Cylinders, and Cones

Surface area is a measurement of the total area of all sides of a 3-dimensional object.

Prisms





A_{base} = triangle $\frac{b \cdot l}{2}$

$A_{rectangles} = ah + bh + ch$

$A_{total} = 2A_{base} + ah + bh + ch$

Surface area of rectangular prisms

$SA = 2(wh + lw + lh)$

Surface area of triangular prisms

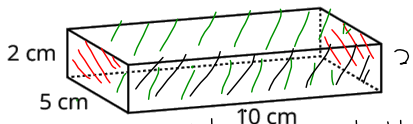
$SA_{total} = 2\left(\frac{bl}{2}\right) + ah + bh + ch$

In ***general***, the surface area of a prism is given by:

$S.A = 2A_{base} + \text{Area lateral}$ side

Example 1: Determine the surface area of each of the following objects.

a)



left + right top + bottom

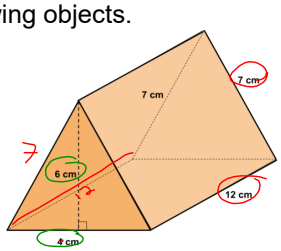
$S.A = 2(2 \times 5) + 2(5 \times 10) + 2(10 \times 2)$

$= 2(10) + 2(50) + 2(20)$

$= 20 + 100 + 40$

$= 160\ cm^2$

b)



$A_{\triangle} = \frac{b \times h}{2}$

$= \frac{4 \cdot 6}{2}$

$= 12\ cm^2$

$A_{sides} = (7 \times 12) + (7 \times 12) + (4 \times 12)$ bottom

$= 84 + 84 + 48$

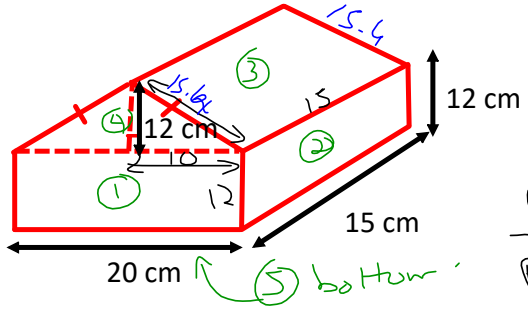
$= 216\ cm^2$

$S.A = 216 + 2 \cdot 12$

$= 216 + 24$

$= 240\ cm^2$

Example 2: Determine the surface area of the following solid.



1848 cm²

Area ②
A = lxw
= 15x12
= 180 cm²

$x^2 = 12^2 + 10^2$
$= 144 + 100$
$= 244$
$x = \pm\sqrt{244}$
$\doteq 15.6$ cm

Area ③
A = lxw
= 15x15.6
= 234 cm²

Area ④
A = bxh / 2
= 20x12 / 2
= 120 cm²

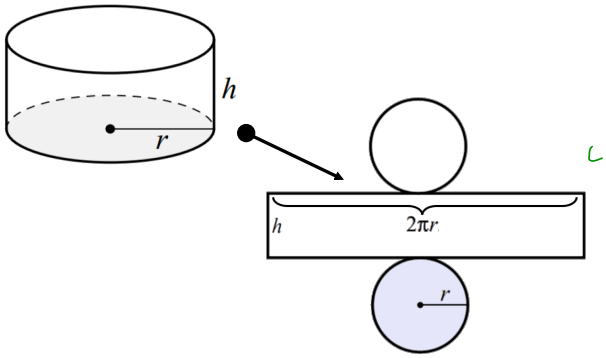
Area ①
A = lxw
= 20x12
= 240 cm²

Area ⑤
A = lxw
= 15x20
= 300 cm²

Total = 2(240) + 2(180) + 2(234) + 2(120) + 300
= 1848 cm²

Cylinders

A **cylinder** is similar to a prism, except that it has a circular base (not a polygon).



$A_{base} = \pi r^2$

$A_{lateral\ surface} = 2\pi rh$

$A_{total} = 2A_{base} + A_{lateral\ surface}$

curved part

Surface area of cylinders

$A_{total} = 2\pi r^2 + 2\pi rh$

Example 3:

a) Determine the amount of paper needed for the label of this can of soup.

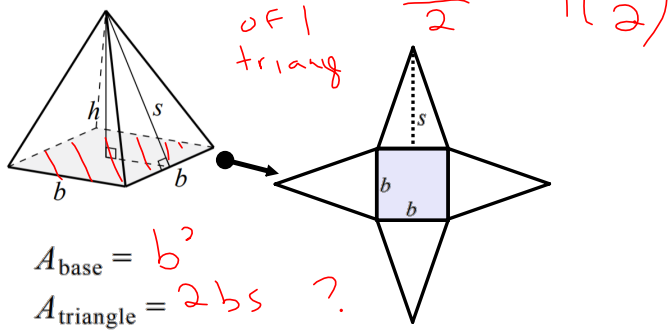
r = 3

A = 2πrh
= 2π(3)(8)
≈ 151 cm²

b) Determine the total area of the metal part of the can.

S.A = 2πr² + 2πrh
= 2π(3)² + 151
= 2π·9 + 151
= 18π + 151
≈ 207.4 cm²

Square-based pyramids and Cones

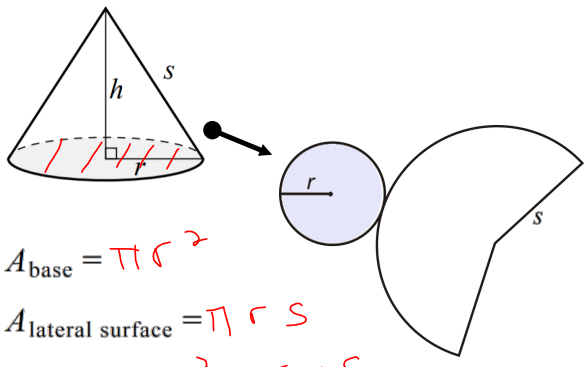


Area of 1 triang $= \frac{bs}{2}$ $4\left(\frac{bs}{2}\right)$

$A_{base} = b^2$

$A_{triangle} = 2bs$?

$A_{total} = b^2 +$



$A_{base} = \pi r^2$

$A_{lateral\ surface} = \pi r s$

$A_{total} = \pi r^2 + \pi r s$

Surface area of square-based pyramids

$A_{total} = b^2 + 2bs$

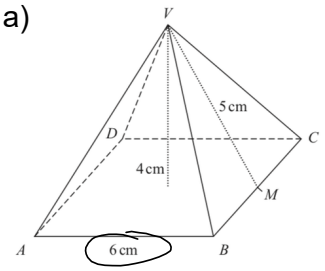
Surface area of cones

$A_{total} = \pi r^2 + \pi r s$

↑ Curved part

Example 4: Determine the surface area of the following objects.

a)



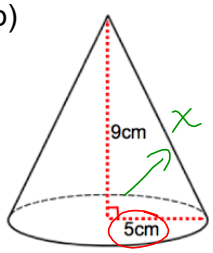
$S.A = b^2 + 2bs$

$= 6^2 + 2(6)(5)$

$= 36 + 60$

$= 96\ cm^2$

b)



$r = 5$

Need slant height

$x^2 = 9^2 + 5^2$

$= 81 + 25$

$= 106 \therefore x > 0$

$x = \pm\sqrt{106}$

$x \doteq 10.3\ cm$

$S.A = \pi r^2 + \pi r s$

$\doteq \pi 5^2 + \pi(5)(10.3)$

$\doteq 246.3\ cm^2$