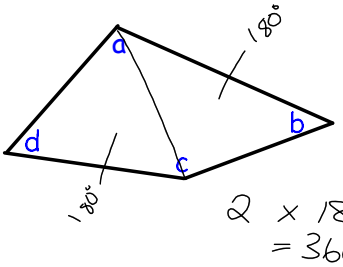
5.4 Angles in Polygons

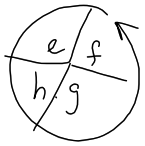
How can we find the sum of the angles in a quadrilateral? Can you think of a way without using a protractor?



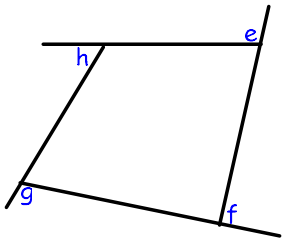
$2 \times 180^\circ = 360^\circ$

Lets put our heads together and figure this out!

Pull



360°

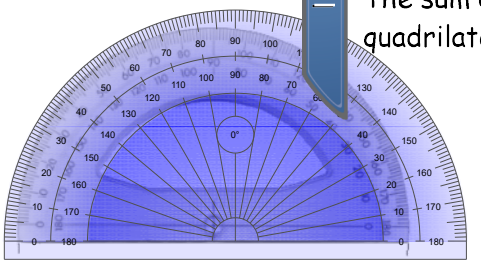


What do you think the exterior angles of a quadrilateral add up to?

Pull

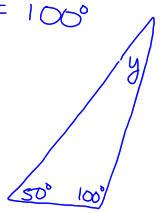
$e + f + g + h = 360^0$

The sum of the exterior angles of a quadrilateral adds up to 360^0

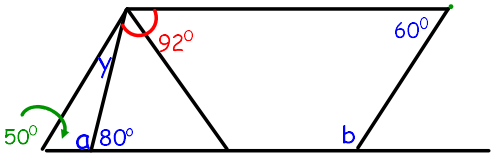


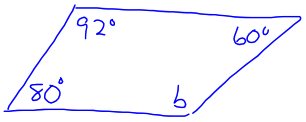
Ex. 1 Find the unknowns

$a = 180^\circ - 80^\circ$ (SAT)
$= 100^\circ$



$y = 180^\circ - 50^\circ - 100^\circ$ (ASTT)
$= 30^\circ$





Interior Angle Theorem (IAT)

$b = 360^\circ - 80^\circ - 92^\circ - 60^\circ$
$= 360 - 202$
$= 158^\circ$

What is the sum of the interior angles of a polygon?
What is the sum of the exterior angles of a polygon?

Use the same strategy to complete the table below.

(diagrams are on next page)

Number of Sides	Polygon Name	Sum of Interior Angles	Sum of Exterior Angles
3	Triangle	180°	360°
4	Quadrilateral	360°	360°
5	Pentagon	540°	360°
6	Hexagon	720°	360°
7	Heptagon	900°	360°
8	Octagon	1080°	360°

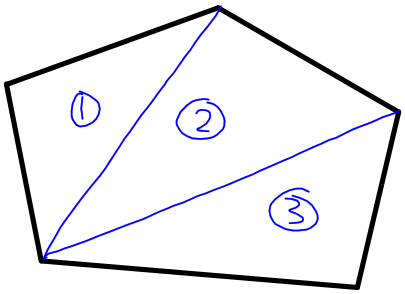
+180 +180

$(n-2) \times 180°$

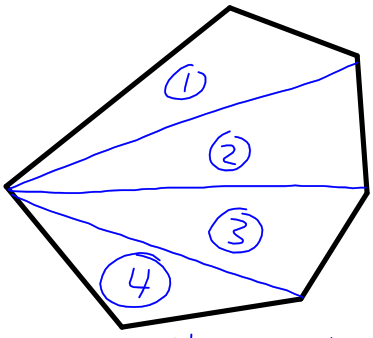
The sum of the exterior angles of a convex polygon is: 360°

Look for a pattern in the sum of the interior angles column. Determine a formula for the sum of the interior angles of a polygon based on the number of sides.

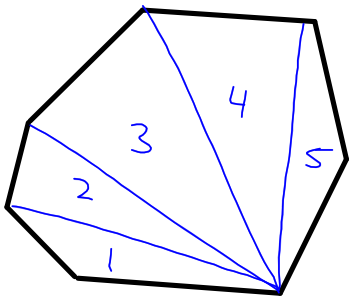
The sum of the interior angles of a polygon with *n* sides is: $(n-2) \times 180°$



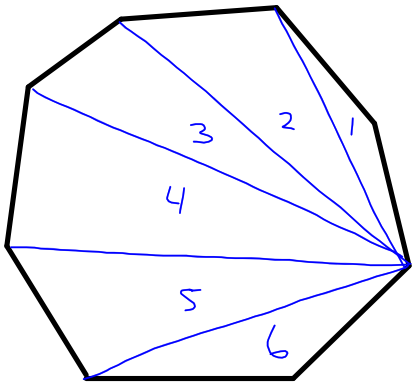
$= 3 \times 180^\circ$
$= 540^\circ$



$= 4 \times 180^\circ$
$= 720^\circ$



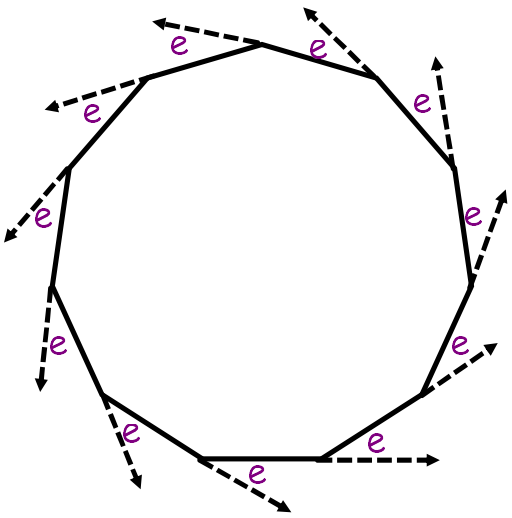
$5 \times 180^\circ$
$= 900^\circ$



$6 \times 180^\circ$
$= 1080^\circ$

Regular polygon: polygon with equal sides and equal interior angles.

Ex. 2 Determine the measure of each exterior angle in a regular 11-sided polygon.



(EAT)

$$\frac{11e}{11} = \frac{360}{11}$$

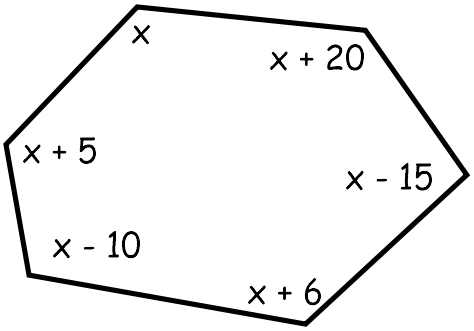
$$e = \frac{360}{11}$$

$$\approx 32.7^\circ$$

Ex. 3 Determine the measure of each interior angle in a regular 15-sided polygon.

METHOD #1	METHOD #2
Sum $= (n-2) \times 180°$ $n = 15$ Sum $= (15-2) \times 180°$ $= 13 \times 180°$ $= 2340°$ $\frac{15i}{15} = \frac{2340°}{15}$ (IAT) $i = \frac{2340}{15}$ $= 156°$	

Ex. 4 Determine the value of *x*.



Sum = $(n-2)180°$

$n = 6$

$\text{Sum} = (6-2)180°$

$= 4(180°)$

$= 720°$

$x + x+20 + x-15 + x+6 + x-10 + x+5 = 720°$

$6x + 6 = 720$

$6x = 720 - 6$

$\frac{6x}{6} = \frac{714}{6}$

$x = \frac{714}{6}$

$= 119°$

Ex. 5 The interior angles of a regular polygon add to 1440°.

How many sides does the polygon have?

Sum = $(n-2)180°$

$\frac{1440}{180} = \frac{(n-2)180°}{180°}$

$\frac{1440}{180} = n-2$

$8 = n-2$

$10 = n$

Pull

$\therefore$

Ex. 6 How many sides does a polygon have if each of its interior angles measures 162°?

Ex 7. In a regular polygon, the ratio of the measure of the exterior angle to the measure of the adjacent interior angle is 1 to 4. How many sides does the polygon have?

Summary

Sum of Interior Angles

Where n is the number of sides of a polygon ⟶

Sum of Exterior Angles

Sum of all exterior angles of any polygon is