3.10 HOMEWORK HANDOUT – SOLVING LINEAR SYSTEMS ALGEBRAICALLY

PART A

1) Determine if the given point is a solution to the given linear system using a **formal check**.

a) Point: $(2,15)$
$y = 2x + 14$
$y = 5x + 1$

b) Point: $(2,22)$
$y = 15x - 8$
$y = 10x + 2$

c) Point: $(-4, 3)$
$y = 2x + 11$
$y = -x - 7$

2) Solve the following linear systems. Write your solution as a coordinate.

a) $y = 2x + 5$
$y = 5x - 4$

b) $C = -5n + 13$
$C = 3n + 5$

c) $P = 15t + 137$
$P = -5t + 97$

d) $y = -8x + 19$
$y = -4x + 7$

e) $V = 6t - 5$
$V = 4t - 1$

f) $y = -3x + 5$
$y = -3 + 5x$

g) $C = -18n + 10$
$C = -5n + 75$

h) $d = 8t + 9$
$d = 3t + 19$

3) An event planner offers the following two pricing options:

Option A: A one-time up-front payment of $600.
Option B: A rate of $40/h, paid after the event.

a) Create an equation for each option. Define your variables!
b) If an event is expected to require 10 hours of planning, which option will result in a lower cost to the customer, and by how much?
c) Explain the meaning of the point where the two lines intersect.
d) Explain why a customer might choose Option A.

4) Two tutors charge according to the following equations, relating the tutoring charge, C, in dollars, to the time, t, in hours:

- **Mr. Wellington:** $C = 40t$
- **Ms. Tenshu:** $C = 35t + 20$

a) Solve the linear system algebraically and explain what the solution means.
b) Under what conditions should a student hire each tutor?

5) The volume of liquid, in litres, in two containers after t minutes is given by the following equations.

Container A: $V = 10t + 50$ **Container B:** $V = 140 - 8t$

a) State the initial volume of liquid in each container.
b) State whether each container is filling or draining.
c) After approximately how many minutes do the containers contain the same volume of liquid? What is the volume?

PART B

6) Samantha's tanker truck carrying fuel initially contains 10 000 L. Fuel is added to the tank at a rate of 2000 litres per minute. Jim's tanker truck initially contains no fuel, but is being refuelled at a faster rate of 3000 litres per minute.

a) Determine an equation to model the volume of fuel in both trucks t minutes after filling begins.
b) Algebraically determine the solution to the linear system using the equations you found in a).
c) What does the solution represent in this scenario?

7) Faster Fitness has a monthly membership fee of $90. Members pay $2.50 to take an aerobics class.
At Drop-in Fitness, there is no membership fee, but clients pay $10 per class.
a) Write a linear relation for the **yearly** cost in terms of the number of aerobics classes.
b) Solve the linear system, what does the point of intersection mean?
c) How would you advise someone who is trying to choose between the two fitness clubs?

8) Marie charges $3 for every 4 bottles of water purchased from her store. She pays her supplier $0.25 per bottle, plus $250 for shelving and water delivery:
a) Create a system of two linear equations to model this situation.
b) Solve the system algebraically.
c) What does the intersection of these two lines represent?

PART C

9) Under what conditions does a linear system have one solution? No solution? Infinitely many solutions? Discuss in terms of rate of change and initial value.

10) Solve the following systems of linear equations using the comparison method.

a) $5y = 2x + 5$	b) $2x = -y - 8$	c) $3x - y + 5 = 0$
$5y = 10x - 35$	$2x = 5y + 22$	c) $y = 2x + 15$

11) Consider the system following system of equations:

$$y = ax + b, \;\; y = cx + d$$

Find a **general solution** for x and y. Are there any restrictions for the values of a, b, c or d?

<u>Helpful hint</u>: $ax - cx = (a - c)x$

ANSWERS

1) a) Not a solution b) It is a solution c) Not a solution

2) a) $(3,11)$ b) $(1,8)$ c) $(-2,107)$ d) $(3,-5)$ e) $(2,7)$ f) $(1,2)$
g) $(-5,100)$ h) $(2,25)$

3) a) Let C be the cost in $ and t the number of hours of planning needed.

$$C = 600, \quad C = 40t$$

b) Option B
c) The point of intersection represents when the cost for both options are equal.
d) A customer might choose Option A if they know their event won't require many hours of planning. The solution to this system is (15, 600) – A customer would choose option A if if their event needed 15 hours or less of planning time.

4) a) (4, 160). The solution represents that both tutors charge the same amount ($160) for the same amount of time (4 hours).
b) If the student needs the tutor for fewer than four hours, they should choose Mr. Wellington. If they need them for more than four hours, they should hire Ms. Tenshu. For exactly four hours, it doesn't matter who they choose.

5) a) Container A: 50L, Container B: 140L
b) Container A is filling (positive RoC), container B is draining (negative RoC).
c) After 5 minutes the containers have the same volume of 100L.

6) a) Let V represent the volume of fuel in the tanker (L) and t represent the time in minutes.

Samantha: $V = 2000t + 10000$, Jim: $V = 3000t$

b) $(t, V) = (10,\ 30000)$
c) The solution to the linear system represents the time (10 minutes) where both tankers have the same volume of 30 000L.

7) a) Let C represent the yearly cost in $ and n be the number of aerobics classes.

Faster Fitness: $C = 2.50n + 1080$ **Drop-in Fitness:** $C = 10n$

b) $(144, 1440)$. The point of intersection means that both gyms have the same cost of $1440 for 144 aerobics classes.
c) If someone was planning to attend fewer than 144 aerobics classes in a year, they should choose Drop-in Fitness. Otherwise, they should choose Faster fitness. At exactly 144 classes, the choice doesn't matter.

8) a) Let C represent the cost in $ and n the number of water bottles sold.

Cost (revenue): $C = 0.75n$ Cost (expenses): $C = 0.25n + 250$

b) $(C, n) = (500, 375)$
c) The intersection represents where Marie's cost for buying water is equal to her revenue – her "break-even" point. Once Marie sells 500 water bottles (for $375), every bottle sold afterwards is purely profit!

9) **One solution:** equations have different rates of change.
No solutions: equations have the same rate of change, but different initial values.
Infinitely many solutions: equations have the same rate of change and initial value.

10) a) $(5,3)$ b) $\left(-\frac{3}{2}, -5\right)$ c) $(10, 35)$

11) General solution: $\left(\frac{d-b}{a-c}, \frac{a(d-b)}{a-c} + b\right)$ or more simply $\left(\frac{d-b}{a-c}, \frac{ad-cb}{a-c}\right)$. The only restriction is that $a \neq c$ or else you would be dividing by zero!