3.8 - Linear Models

Linear relations are defined by two main components.

1) The **constant of variation** or rate of change (RoC) ⟵ How do things change?

2) The **constant value** or initial value ⟵ What value does it start at?

Example 1: You're planning a large event for friends and family at a local venue. The venue charges a **fixed** cost of $150 to rent their space. Their charge an additional $6 **per guest** for food and drink.

a) How do you find the cost for 10 guests?

$$Cost = 150 + 6(10)$$
$$= 150 + 60$$
$$= 210$$
∴ It will cost $210

b) How do you find the cost for 50 guests?

$$Cost = 150 + 6(50)$$
$$= 150 + 300$$
$$= 450$$
∴ it will cost $450

What is the same for a) and b)? What changes?

⟶ The fixed cost ($150) stays the same. The number of guests changes (variable).
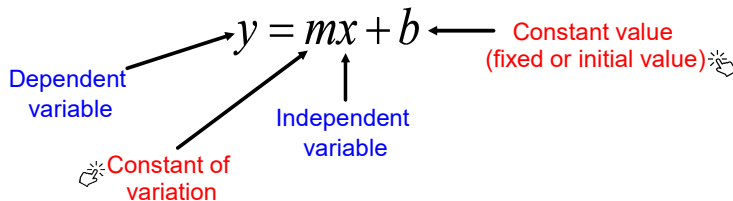
c) How do you find the cost for "n" guests?

$$C = 6n + 150$$

Cost per gurst   Fixed rate
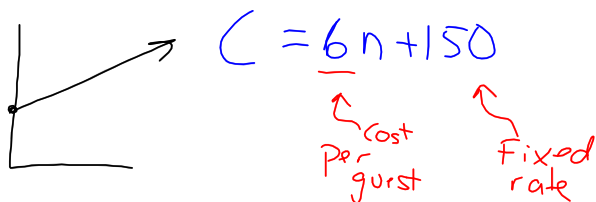
⟵ This gives us a linear relationship that models this situation!

The **constant of variation** in our relation is __$6/guest__. It represents a **unit rate.**

The **constant value** in our relation is __$150__. It represents the **fixed amount.**

Linear relationships can be written in the following way:

$$y = mx + b$$

Dependent variable → y

Constant of variation → m

Independent variable → x

Constant value (fixed or initial value) → b

Example 2: Rim opens a savings account and starts off with $45. They plan to put aside $25 each month into this account.

a) Determine a equation that models how much Rim has in their bank account over time.
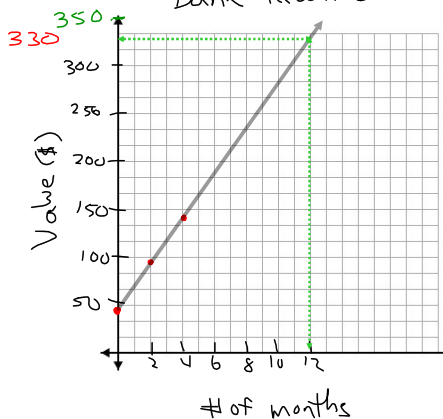
$$V = 25n + 45$$

Let n represent # of months

b) Fill out a table of values for 6 months.

n	V	
0	45	↳+25
1	70	↳+25
2	95	↳
3	120	↳
4	145	↳
5	170	
6	195	

c) Use your table of values to graph the relationship.

Bank Account Over time

Value ($) vs # of months

350, 330

d) Use your graph to estimate how much money Rim will have after one year.
Use dotted lines on the graph.

About $330

e) Use your equation to verify your answer from d).

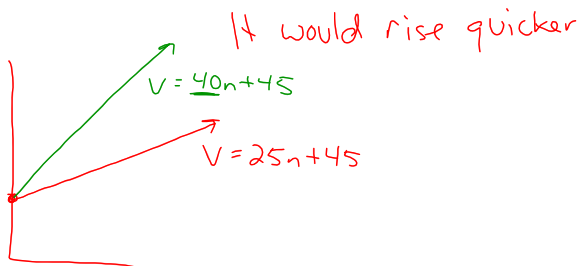*Don't forget brackets when substituting.*

$$V = 25n + 45$$
Sub $n = 12$
$$V = 25(12) + 45$$
$$= 345$$

∴ The amount is exactly $345

f) How would your graph change if Rim saved more money per month?

It would rise quicker

$V = 40n + 45$

$V = 25n + 45$

Example 3: Jason won a $125 prize in a contest and immediately put it into their bank account. With the money, Jason starts a daily habit of buying a specialty coffee at Garbucks for $3.50.

a) What is the constant value? What is the constant of variation?
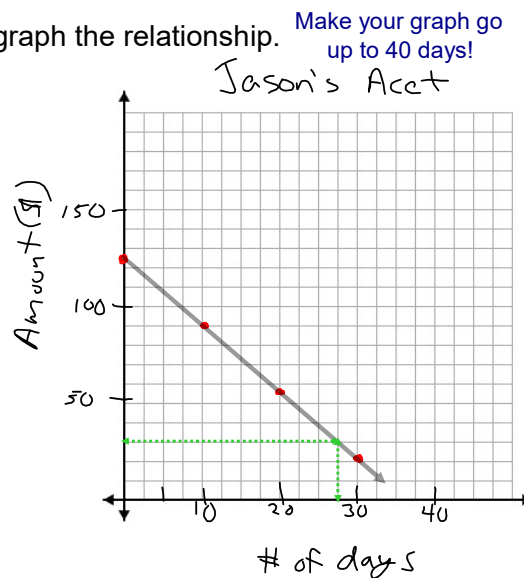
$125 $-3.50/\text{day}$

b) Determine an equation that models how much money Jason has in their account over time.

$A = 125 - 3.50n$
OR
$A = -3.50n + 125$

Let n represent # of days

c) Fill out a table of values and use it to graph the relationship.   Make your graph go up to 40 days!

I   D

n	A
0	125
10	90
20	55
30	20
40	?

$\}-35$
$\}-35$
$\}$

Jason's Acct

Amount ($) — # of days

d) Jason has $30.50 left in their bank account. Use the graph to **estimate** how many days they have been buying coffee.

About 28 days

e) Verify your answer in d) algebraically to determine the **exact** number of days.

$A = 125 - 3.50n$
Sub $A = 30.50$

$30.50 = 125 - 3.50n$
$30.50 - 125 = -3.50n$
$\dfrac{-94.50}{-3.50} = \dfrac{-3.50n}{-3.50}$

$27 = n$

∴ It would take 27 days

Example 4: Find the equation that represents each of the following tables of values.

'x' 'y'

a)

n	C
0	6
1	9
2	12
3	15
4	18

START!

) +3

) +3

$C = 3n + 6$

b)

x	y
0	25
1	19
2	13
3	7
4	1

) -6

) -6

$y = -6x + 25$

c)

0   7

n	P
1	16
2	25
3	34
4	42
5	51

) +9

) +9

$P = 9n + 7$