2.11 Solving Proportions

Proportion: A statement of equality between ratios.

Ex. 4:5 = 12:15 or $\frac{4}{5}=\frac{12}{15}$ or $\frac{4}{12}=\frac{5}{15}$

Ex. 1 Solve each proportion.

a) 3:4 = 6:x

×2 ×2

$x = 8$

b) x:7 = 30:21

÷3 ÷3

$x = 10$

c) y:3 = 4:10

Set up as a fraction!

$\frac{y}{3} = \frac{4}{10}(3)$

$y = \frac{12}{10}$

$= \frac{6}{5}$

This is not obvious. For more complex proportions you can solve using algebra.

Let's look for a pattern:

$\frac{x}{2} = 4$ (×2)

$x = 8$

$5 = \frac{x}{3}$ (×3)

$15 = x$

$\frac{8}{x} = \frac{4}{3}$

(×x) $8 = 4(x)$

$8 = 4x$

$2 = x$

$\frac{a}{b} = \frac{c}{d}$

$a \cdot d = b \cdot c$

This is called "cross-multiplication"

d) 2:3 = 15:x

$\frac{2}{3} = \frac{15}{x}$

$2x = 45$

$x = \frac{45}{2}$

Cross-multiplying is very useful when the variable is in the denominator.

e) 7: x = 8 : 15

$\frac{7}{8} = \frac{x}{15}$

$\frac{7}{x} = \frac{8}{15}$

$105 = 8x$

$\frac{105}{8} = x$

f) 4: 17 = 3 : 2k

$\frac{4}{17} = \frac{3}{2k}$

$8k = 51$

$k = \frac{51}{8}$

Ratios can compare more than 2 terms.

g) 5:6:7 = 15:18:x

×3 ×3 ×3

$x = 7 \times 3$

$= 21$

h) 5:8:11 = a:b:30

$\frac{5}{a} = \frac{8}{b} = \boxed{\frac{11}{30}}$

Work with 2 fractions at a time!

$\frac{11}{30} = \frac{5}{a}$

$11a = 150$

$a = \frac{150}{11}$

$\frac{11}{30} = \frac{8}{b}$

$11b = 240$

$b = \frac{240}{11}$

Ex. 2 To make a punch you need 180 mL of gingerale and 150 mL of apple juice. Haifa has a 200 mL container of apple juice. How much gingerale should she add to make the same recipe of punch?

G : A = G : A

180 : 150 = x : 200

$\frac{180}{150} = \frac{x}{200}$

$150x = 36000$

$\frac{150x}{150} = \frac{36000}{150}$

$x = 240$

∴ Haifa needs 240mL of gingerale.

Ex. 3 If 12 out of 15 students prefer chocolate ice cream over vanilla, how many students prefer chocolate in a group of 200?

choc : total = choc : total

12 : 15 = c : 200

$\frac{12}{15} = \frac{c}{200}$

$15c = 2400$

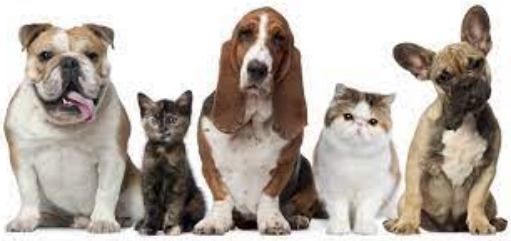
$\frac{15c}{15} = \frac{2400}{15}$

Total

$c = 160$

∴ 160 students prefer chocolate over vanilla out of 200.

Ex. 4 There are 600 cats and dogs in a shelter. The ratio of cats to dogs in this shelter is 3:5. How many cats and how many dogs are in this shelter?



c : d : total c : d : total

3 : 5 : 8 = c : d : 600

×75 ×75 ×75

$\frac{600}{8} = 75$

$c = 3 \times 75 = 225$

$d = 5 \times 75 = 375$

∴ There are 225 cats and 375 dogs.

Ex. 5 Hanna scored 18 points in her last basketball game. If she scored 29% of the total team points, how many points did her team score in the game?

Hanna's points : total. Hanna's points : total.

$29:100 = 18:x$

$\frac{29}{100} = \frac{18}{x}$

$\frac{29x}{29} = \frac{1800}{29}$

$x = 62$

∴ Hanna's team scored a total of 62 points.

Ex. 6 The Bouraoui family bought a $425,000 house with a $125,000 down payment. They are also buying a $32,000 car and want to make a down payment for the car that is the same proportion as the one they made for the house. How much should their down payment on the car be?

House down : Total = Car down : Total

$125{,}000 : 425{,}000 = x : 32{,}000$

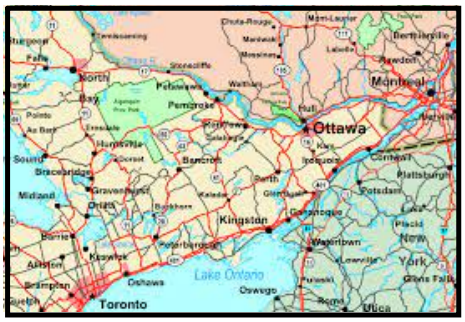
$\frac{125{,}000}{425{,}000} = \frac{x}{32{,}000}$

$\frac{425{,}000x}{425{,}000} = \frac{4\,000\,000}{425{,}000}$

$x = 9411.76$

∴ The car down payment is $9411.76.

Ex. 7 A map measures 6 cm from Ottawa to Kingston a distance of 196 km. The distance from Ottawa to North Bay measures 11 cm on the same map. What is the actual distance from Ottawa to North Bay?



Map : distance

$6:196 = 11:x$

$\frac{6}{196} = \frac{11}{x}$

$\frac{6x}{6} = \frac{2156}{6}$

$x = 359.3$ or $x = 359.3\overline{3}$

∴ The distance is about 359.3 km.