

2.4 Exponent Laws - Day 2

Ex. 1 Simplify.

RECALL

$$\begin{aligned} a) (2^2)^4 &= 2^{2 \times 4} \\ &= 2^8 \\ &= 256 \end{aligned}$$

$$\begin{aligned} b) (x^3)^2 &= x^{3 \times 2} \\ &= x^6 \end{aligned}$$

 **Raising a Power to an Exponent**

To raise a power to another exponent, multiply the exponents without changing the base.

Power of a Power Rule

$$(m^a)^b = m^{a \times b}$$

Ex. 2 Simplify.

$$\begin{aligned} a) (2x^5)^4 &= (2x^5)(2x^5)(2x^5)(2x^5) \\ &= 2^4 (x^5)^4 \\ &= 16x^{20} \end{aligned}$$

$$\begin{aligned} b) (-2xy^2)^3 &= (-2)^3 x^3 (y^2)^3 \\ &= -8x^3 y^6 \end{aligned}$$

 **Power of a Product**

The exponent is applied to each part of the base.

$$(ab)^m = a^m b^m$$

Ex. 3 Simplify.

$$\begin{aligned} a) \left(\frac{2}{3}\right)^3 &= \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) \\ &= \frac{2^3}{3^3} \\ &= \frac{8}{27} \end{aligned}$$

$$\begin{aligned} b) \left(\frac{5}{x}\right)^2 &= \frac{5^2}{x^2} \\ &= \frac{25}{x^2} \end{aligned}$$

 **Power of a Quotient**

The exponent is applied to each part of the base.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Ex. 4 Simplify.

$$\begin{aligned} a) (-4x^2y)^4 &= (-4)^4 (x^2)^4 y^4 \\ &= 256x^8y^4 \\ &= y^{-3-(-14)} \\ &= y^{-3+14} \end{aligned}$$

$$\begin{aligned} b) \frac{(2x^3y^1)^3}{(-2x^2y^7)^2} &= \frac{2^3 x^9 y^3}{(-2)^2 x^4 y^{14}} \\ &= \frac{8x^9 y^3}{4x^4 y^{14}} \\ &= 2x^5 y^{-11} \end{aligned}$$

$$\begin{aligned} c) (-x^5y^2)^3(3x^2y^3)^2 &= (-1)^3 x^{15} y^6 (3^2 x^4 y^6) \\ &= (-x^{15} y^6) (9 x^4 y^6) \\ &= -9 x^{19} y^{12} \\ &= -9x^9 (1) \\ &= -9x^9 \end{aligned}$$

Exponent Laws

Multiply powers

$$\begin{aligned} m^3 \cdot m^4 \\ = m^{3+4} \\ = m^7 \end{aligned}$$

→ add exponents

Divide powers

$$\begin{aligned} m^6 \div m^1 \\ = m^{6-1} \\ = m^5 \end{aligned}$$

→ subtract exponents

Power of a power

$$\begin{aligned} (m^6)^3 \\ = m^{6 \times 3} \\ = m^{18} \end{aligned}$$

→ multiply exponents

Multiplication

$$\begin{aligned} 3m^2 \cdot 4m^5 \\ = 12m^7 \end{aligned}$$

(3 and 4 are grouped together)

→ Multiply the coefficients

Division

$$\begin{aligned} 50m^8 \div 2m^3 \\ = 25m^5 \end{aligned}$$

→ Divide the coefficients

Power of a Power

$$\begin{aligned} (4m^3)^2 \\ = 4^2 m^6 \\ = 16m^6 \end{aligned}$$

→ Exponent affects each part of the base

Ex. 5 Simplify.

$$\text{a) } \overbrace{(a^2 b^3)}^4$$

$$= a^8 b^{12}$$

$$\text{b) } (-4m^2)^3$$

$$= (-4)^3 m^6$$

$$= -64 m^6$$

$$\text{c) } \overbrace{(-x^3)}^2 \overbrace{(2x^4)}^3$$

$$= ((-1)^2 x^6) (2^3 x^{12})$$

$$= (x^6)(8x^{12})$$

$$= 8x^{18}$$

$$\text{d) } \frac{(5c^3d)(4c^2d^2)}{(2c^2d)^2}$$

$$= \frac{20c^5d^3}{2^2 c^4 d^2}$$

$$= \frac{20c^5d^3}{4c^4 d^2}$$

$$= 5cd$$

$$\text{e) } \frac{(-3m^2n^6)(2m^4n^8)^3}{(4mn^2)^3}$$

$$= \frac{(-3m^2n^6)(2^3 m^{12} n^{24})}{4^3 m^3 n^6}$$

$$= \frac{(-3m^2n^6)(8m^{12} n^{24})}{64m^2 n^6}$$

Reduce

$$\frac{-24}{64} = \frac{-24}{64} \boxed{\frac{m^{14} n^{30}}{m^2 n^6}}$$

$$= \frac{-3}{8} m^{12} n^{24}$$

$$= -\frac{3}{8} m^{12} n^{24}$$

$$\text{f) } \frac{(2m^3)^4}{24m^5}$$

$$= \frac{2^4 m^{12}}{24 m^5}$$

$$= \frac{16 m^{12}}{24 m^5}$$

$$= \frac{2 m^7}{3}$$

$$= \frac{2}{3} m^7$$