1.3 Number Sets, Density and Limits

History of Zero - Kyne Santos

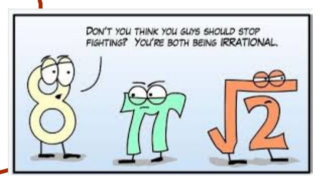
Recall:
- N= set of natural numbers N={1,2,3,4,...}
- W= set of whole numbers W= {0,1,2,3,4,...}
- Z= set of integers Z= {...-2,-1,0,1,2,3,...}

Rational Numbers (Q)
- can be written as a fraction
- includes repeating decimals
- includes terminating decimals

π $\sqrt{2}$

Irrational Numbers (Q' or P)
- can NOT be written as a fraction "Q prime"
- includes non-terminating non-repeating decimals

REAL Numbers (R)
- includes all rational and irrational numbers
- does not include imaginary* numbers



Ex. 1 State all sets (N,W,Z,Q, Q' or P) that each number belongs to.

a) 5 N Z W Q

b) 0 W Z Q

c) -3 Z Q

d) 5.8 Q

e) -6.7 Q

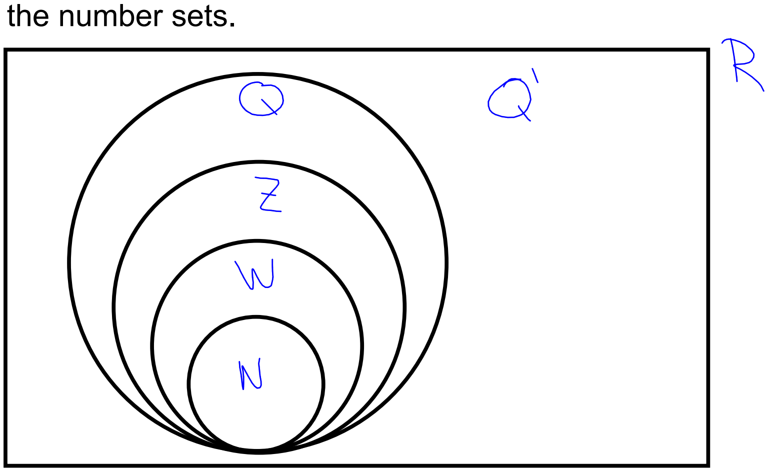
f) 4.15151515... $= 4.\overline{15}$ Q

g) $\frac{-7}{10}$ Q

h) 0.22222... $= 0.\overline{2}$ Q

k) $\sqrt{2}$ and π Q'

Ex. 2 Label the Venn Diagram to show the relationship between the number sets.



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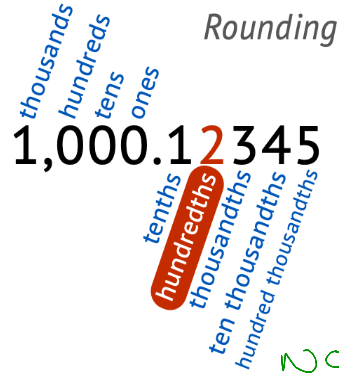

NOTE
Look at column of our last digit

Ex. 3 Write the following terminating decimals as fractions.

a) 0.7
$= \frac{7}{10}$

b) -0.75
$= -\frac{75}{100} \begin{matrix}\div 25\\ \div 25\end{matrix}$
$= -\frac{3}{4}$

c) 0.125
$= \frac{125}{1000} \begin{matrix}\div 125\\ \div 125\end{matrix}$
$= \frac{1}{8}$

d) 4.57
$= 4\frac{57}{100}$

e) -12.6
$= -12\frac{6}{10}$
$= -12\frac{3}{5}$

f) -8.4213789
$= -8\frac{4213789}{10{,}000{,}000}$

g) 0.000000023
$= \frac{23}{1{,}000{,}000{,}000}$

Notice anything?

$$0.\overline{18} = \frac{18}{99} = \frac{2}{11}$$
$$0.\overline{123} = \frac{123}{999} = \frac{41}{333}$$
$$0.\overline{8765} = \frac{8{,}765}{9{,}999}$$

Ex. 4 Write the following repeating decimals as fractions.

a) 0.7777...
$= 0.\overline{7}$
$= \frac{7}{9}$

b) -0.33...
$= -0.\overline{3}$
$= -\frac{3}{9} \begin{matrix}\div 3\\ \div 3\end{matrix}$
$= -\frac{1}{3}$

c) 8.66...
$= 8.\overline{6}$
$= 8\frac{6}{9}$
$= 8\frac{2}{3}$

d) -32.1818...
$= -32.\overline{18}$
$= -32\frac{18}{99} \begin{matrix}\div 9\\ \div 9\end{matrix}$
$= -32\frac{2}{11}$

e) 10.$\boxed{0236}$0236...
$= 10.\overline{0236}$
$= 10\frac{236}{9999}$

Limit Property

- When a sequence or pattern of numbers gets closer and closer to a single number. That number is called the **limit**.

Ex. 5 Find the limit for each sequence (if it exists)

8.1 8.01 8.001 ... 8

a) 8.1, 8.01, 8.001, 8.0001, 8.00001, 8.000001,... 8

b) 4.7, 4.77, 4.777, 4.7777, ... $4.\overline{7}$

c) -12.65, -12.$\boxed{65}$65, -12,656565, -12.65656565,... $-12.\overline{65}$

d) 2,4,8,16,32,64,... ∞ (NO LIMIT)

e) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ $\frac{1}{\text{HUGE \#}}$ Limit Ø

f) 1,4,9,16,25,36,... ∞ NO LIMIT

g) 32,16,8,4,2,... $\div 2$ $\div 2$ $\div 2$ $\div 2$

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

$\frac{1}{\text{HUGE \#}}$ Limit Ø