1.8 HOMEWORK: EXPONENT LAWS

PART A

1) Show why the following is true:

a) $2^4 \times 2^3 = 2^7$ b) $\frac{2^8}{2^5} = 2^3$.

2. Show why the following is true:

a) $\left(2^3\right)^4 = 2^{12}$ b) $(2\times 3)^4 = 2^4 \times 3^4$

3) State the value that should be placed in each box.

a) $2^3 \times 2^4 = 2^{\square}$ b) $\left(x^5\right)\left(x^2\right) = x^{\square}$ c) $5\left(5^{11}\right) = 5^{\square}$

d) $\left(10^6\right)\left(10^2\right)(10) = 10^{\square}$ e) $(-6)^7(-6)^8 = \square^{15}$

4) State the value that should be placed in each box.

a) $3^9 \div 3^4 = 3^{\square}$ b) $\left(2^3\right)^5 = 2^{\square}$ c) $\left(3^4\right)^6 = \square^{24}$ d) $\frac{6^{15}}{6^{11}} = \square^4$

e) $\frac{2^{18}}{\square^5} = 2^{13}$ f) $\left(\frac{5}{6}\right)^7 = \frac{5^7}{6^{\square}}$

5) Express each of the following as a single power.

a) $5^2 \times 5^{10}$ b) $\left(2.4^3\right)\left(2.4^8\right)$ c) $\left(x^{15}\right)\left(x^3\right)$ d) $1.5\left(1.5^{12}\right)$ e) $m^3\left(m^6\right)$

f) $\left(\frac{2}{3}\right)^4\left(\frac{2}{3}\right)^6$ g) $\left(7^5\right)\left(7^3\right)\left(7^4\right)$ h) $a\left(a^9\right)\left(a^2\right)$ i) $\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)^5\left(\frac{1}{6}\right)^7\left(\frac{1}{6}\right)^3$

6) Express each of the following as a single power.

a) $15^{14} \div 15^6$ b) $\frac{(-8)^{10}}{(-8)^3}$ c) $\frac{b^{13}}{b^4}$ d) $\left(\frac{3}{7}\right)^{10} \div \left(\frac{3}{7}\right)^4$ e) $\frac{3.78^9}{3.78^5}$

f) $\frac{\left(\frac{1}{3}\right)^7}{\left(\frac{1}{3}\right)^3}$ g) $\left(\frac{5}{6}\right)^7 = \frac{5^7}{6^{\square}}$

PART B

7) Simplify.

a) $\frac{\left(5^8\right)\left(5^9\right)}{5^7}$ b) $\frac{\left(\frac{5}{6}\right)^{17}}{\left(\frac{5}{6}\right)^8\left(\frac{5}{6}\right)^4}$ c) $\frac{(-6)^{12}}{-6(-6)^2(-6)^3}$ d) $\frac{\left(\frac{7}{8}\right)^{15} \div \left(\frac{7}{8}\right)^8}{\left(\frac{7}{8}\right)^2\left(\frac{7}{8}\right)^3}$ e) $\frac{4.2^{13}}{4.2^5} \div \frac{4.2^8}{4.2^3}$

8) Simplify and evaluate.

a) $\frac{7^8}{7^6}$ b) $\frac{(3^6)(3^5)}{3^8}$ c) $\frac{(4^{10})(4^2)}{(4^4)(4^5)}$ d) $\frac{5.8^{11}}{(5.8^6)(5.8^4)}$ e) $\frac{\left(\frac{2}{3}\right)^2\left(\frac{2}{3}\right)^5}{\left(\frac{2}{3}\right)^4\left(\frac{2}{3}\right)}$

9) a) Express 4^3 as a power with a base of 2. b) Express 27^2 as a power with a base of 3.

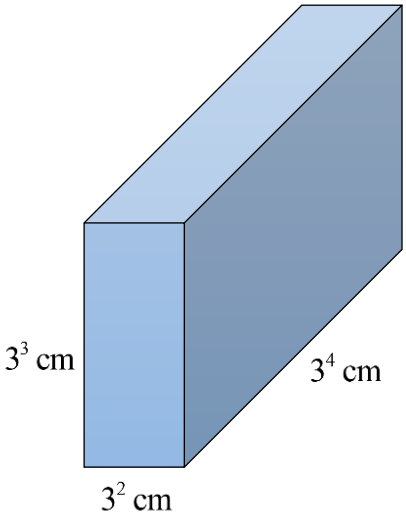
c) Express $\left(\frac{1}{81}\right)^4$ as a power with a base of $\frac{1}{3}$.

d) Without actually calculating the value of either power, show that 16^3 is equal to 4^6.

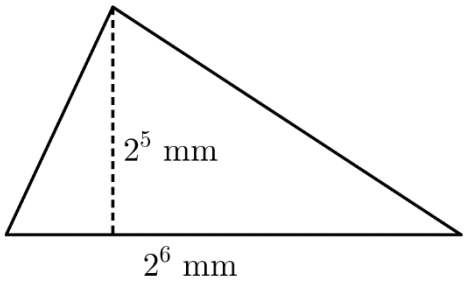
10) The product of two powers is 5^{12}. The quotient of the same two powers is 5^6. Find the two powers.

PART C

11) Determine the volume of the rectangular prism shown on the right. Express your answer as a power.



12) A container has a volume of 5^6 cm^3. A smaller container has a volume of 5^4 cm^3. How many times larger is the volume of the bigger container than that of the smaller container?



13) Determine the area of the triangle shown on the left. Express your answer as a power.

14) Have you ever wondered what it would mean to have an exponent of zero? You can use exponent laws to figure it out!

a) Determine the value of $\frac{2^3}{2^3}$ by first calculating the value of 2^3 and then dividing.

b) Evaluate $\frac{2^3}{2^3}$ using an exponent law. Express your answer as a power.

c) Based on your results from parts (a) and (b), what is the value of 2^0?

d) Use similar reasoning to determine the value of 5^0.

e) Can you think of any cases where this reasoning would fail? Explain.

f) Evaluate each of the following.

i) 3^0 ii) 7^0 iii) 12^0 iv) 0^0 v) a^0

15. Kendra needs to quickly determine whether 3^{40} is greater than or less than 4^{30}, but she does not have access to a calculator. How can she use her knowledge that $3^4 = 81$ and $4^3 = 64$ to solve her problem. Which power has the greater value?

ANSWERS

1) a) One possible explanation is as follows:

$$2^4 \times 2^3 = (2\times2\times2\times2)\times(2\times2\times2)$$
$$= 2\times2\times2\times2\times2\times2\times2$$
$$= 2^7$$

b) One possible explanation is as follows:

$$\frac{2^8}{2^5} = \frac{2\times2\times2\times\cancel{2\times2\times2\times2\times2}}{\cancel{2\times2\times2\times2\times2}}$$
$$= 2\times2\times2$$
$$= 2^3$$

2a)
$$\left(2^3\right)^4 = 2^3\times2^3\times2^3\times2^3$$
$$= (2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)$$
$$= 2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2$$
$$= 2^{12}$$

2b)
$$\left(\frac{2}{3}\right)^4 = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}$$
$$= \frac{2\times2\times2\times2}{3\times3\times3\times3}$$
$$= \frac{2^4}{3^4}$$

3) a) 7 b) 7 c) 12 d) 9 e) –6

4) a) 5 b) 15 c) 3 d) 6 e) 2 f) 7

5) a) 5^{12} b) 2.4^{11} c) x^{18} d) 1.5^{13} e) m^9 f) $\left(\frac{2}{3}\right)^{10}$ g) 7^{12} h) a^{12} i) $\left(\frac{1}{6}\right)^{17}$

6) a) 15^8 b) $(-8)^7$ c) b^9 d) $\left(\frac{3}{7}\right)^6$ e) 3.78^4 f) $\left(\frac{1}{3}\right)^4$

7) a) 5^{10} b) $\left(\frac{5}{6}\right)^5$ c) $(-6)^6$ d) $\left(\frac{7}{8}\right)^2$ e) 4.2^3

8) a) 7^2 ; 49 b) 3^3 ; 27 c) 4^3 ; 64 d) 5.8^1 ; 5.8 e) $\left(\frac{2}{3}\right)^2$; $\frac{4}{9}$

9) a) 2^6 b) 3^6 c) $\left(\frac{1}{3}\right)^{16}$ d) $16^3 = \left(4^2\right)^3$
$= 4^6$

10) 5^9 and 5^3 **11)** 3^9 cm^3 **12)** 25 times larger **13)** 2^{10} mm^2

14) a) 1 b) 2^0 c) 1 d) 1

e) Using this reasoning with a power that has a base of zero would fail, since it would lead to division by zero, which is not defined.

f) i) 1 ii) 1 iii) 1 iv) undefined

v) 1, as long as *a* is a non-zero real number. Undefined if *a* is 0.

15) 3^{40} can be expressed as $\left(3^4\right)^{10}$. 4^{30} can be expressed as $\left(4^3\right)^{10}$. Since Kendra knows that $3^4 > 4^3$, she can conclude that $\left(3^4\right)^{10} > \left(4^3\right)^{10}$. Therefore, 3^{40} is greater than 4^{30}