1.3 HOMEWORK HANDOUT: NUMBER SETS

PART A

1) List the set or sets to which each number belongs ($\mathbb{N},\mathbb{W},\mathbb{Z},\mathbb{Q},\mathbb{R},\mathbb{Q}'$).

     a) $\sqrt{47}$ b) $\frac{14}{2}$ c) -66 d) $\frac{2}{7}$

     e) 3.7 f) π g) $-\sqrt{4}$ h) $\sqrt{2}$

2) State the number of elements (or members) in each set.

     a) {blue, green, red, yellow}   b) {a,b,c,d,e,f,g}   c) {Ontario, Alberta, Manitoba}

     d) $\{2,5,6,8,10\}$ e) $\{-9,-8,-7\}$ f) $\{0,1\}$ g) $\{0\}$ h) $\{1,2,3,4,5,...\}$ i) $\{\ \}$

3) Explain why the set $\{7,9,12\}$ is a subset of the set $\{5,6,7,8,9,10,11,12\}$.

4) Explain why the set $\{0,1,4,9,16\}$ is not a subset of the set $\{1,2,3,4,5,...\}$.

5) Match each parent set on the left with its corresponding subset on the right.

     a) $\{-8,-7,-6,-5,-4,-3,-2,-1\}$ i) $\{1,3,5,7,9\}$

     b) $\{-8,-6,-4,-2,0,2,4,6,8,...\}$ ii) $\{-13\}$

     c) $\{1,2,3,4,5...\}$ iii) $\{0,2,4,6,8,...\}$

     d) $\{...,-9,-7,-5,-3,-1\}$ iv) $\{-1,-2,-3\}$

6) Express the following sets in braces, { }.
     a) The set of natural numbers less than 10.
     b) The set of odd integers from -5 to 5.
     c) The set of all whole numbers.
     d) The set of all integers.
     e) The set of all even whole numbers greater than or equal to 20.
     f) The set of all integers that are multiples of 5.

7) State three numbers between 0 and 1.

8) State four rational numbers between 3 and 4.

PART B

9) List all of the subsets of the set $\{1,2\}$.

10) List all of the subsets of the set $\{-1,0,1\}$.

11) Identify each of the following statements as true or false.
   a) The set of whole numbers is a subset of the set of real numbers.
   b) The set of natural numbers is a subset of the set of integers.
   c) The set of whole numbers is a subset of the set of natural numbers.
   d) The set of rational numbers is a subset of the set of real numbers.
   e) The set of irrational numbers is a subset of the set of rational numbers.
   f) The set of integers is a subset of the set of rational numbers.
   g) $\{2, 3, \pi\}$ is a subset of the rational numbers.

12) Jonah stated that subsets always have fewer elements than their parent sets.
   Is Jonah's claim correct? Explain.

13) The set of real numbers and its subsets are often represented using the symbols shown on the right. Describe how each of the following pairs of sets are related.

   a) $\mathbb{N}$ and $\mathbb{W}$

   b) $\mathbb{W}$ and $\mathbb{Z}$

   c) $\mathbb{Z}$ and $\mathbb{Q}$

   d) $\mathbb{Q}$ and $\mathbb{P}$

Set	Symbol
Natural Numbers	$\mathbb{N}$
Whole Numbers	$\mathbb{W}$
Integers	$\mathbb{Z}$
Rational Numbers	$\mathbb{Q}$
Irrational Numbers	$\mathbb{P}$ or $\mathbb{Q}'$ or $\mathbb{R} \setminus \mathbb{Q}$
Real Numbers	$\mathbb{R}$

14) If set B is a subset of set A, and set C is a subset of set B, is set C a subset of set A. Explain.

15) a) How many natural numbers are there from 1 through 10?
   b) How many whole numbers are there from 0 through 10?
   c) How many rational numbers are there from 1 through 10?
   d) How many irrational numbers are there from 1 through 10?
   e) How many real numbers are there from 1 through 10?

16) Which of the following sets have the density property?
   - natural numbers
   - integers
   - irrational numbers
   - whole numbers
   - rational numbers
   - real numbers

17) Does the set of even integers have the density property? Explain.

18) The following sequences of numbers each have a *limit*. That is, they gradually get closer and closer to a specific number, called the *limit*. Identify the limit of each of the following sequences.

   a) $7.1, 7.01, 7.001, 7.0001, \ldots$

   b) $\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{4}, \dfrac{1}{5}, \dfrac{1}{6}, \ldots$

   c) $\dfrac{5}{1}, \dfrac{5}{4}, \dfrac{5}{9}, \dfrac{5}{16}, \dfrac{5}{25}, \ldots$

   d) $0.3, 0.33, 0.333, 0.3333, \ldots$ e) $-2.6, -2.66, -2.666, -2.6666, \ldots$ f) $1.4, 1.44, 1.444, 1.4444, \ldots$

19) The symbol $\subseteq$ is often used to denote a subset. For example, if set P is a subset of set Q, we would write $P \subseteq Q$. If $A = \{1,2,3,4,5,6,7,8,9\}$, $B = \{2,4,6,8\}$ and $C = \{4,8\}$, indicate whether each of the following statements is true or false.

     a) $A \subseteq B$ b) $B \subseteq A$ c) $C \subseteq B$ d) $C \subseteq A$ e) $B \subseteq C$

20) There is a relationship between the number of elements in a set and the number of possible subsets.
     a) Determine the total number of possible subsets for the set $\{1, 2, 3\}$.
     b) Determine the total number of possible subsets for the set $\{1, 2, 3, 4\}$.
     c) Hypothesize a rule for finding the number of subsets for a set containing n elements.
     d) Use your hypothesis to predict the number of subsets for a set with 12 elements.

21) Consider the set $\left\{1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, ...\right\}$.

     a) Describe how the sequence of numbers in this set is constructed.
     b) Does this set have the density property? Explain.
     c) Does the sequence in this set have a limit? If so, what is the limit? If the sequence does not have a limit, explain why not.

22) Determine the limit of each sequence.

     a) $0.2,\ 0.25,\ 0.252,\ 0.2525,\ ...$ b) $3.128,\ 3.128128,\ 3.128128128,\ ...$

     c) $\sqrt{2}, \sqrt{2 \times \sqrt{2}}, \sqrt{2 \times \sqrt{2 \times \sqrt{2}}}, \sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{2}}}}, ...$

ANSWERS

1) a) $\mathbb{Q}, \mathbb{R}$ b) $\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ c) $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$ d) $\mathbb{Q}, \mathbb{R}$ e) $\mathbb{Q}, \mathbb{R}$ f) $\mathbb{P}$ or $\mathbb{Q}'$ g) $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$ h) $\mathbb{P}$ or $\mathbb{Q}'$

2) a) 4   b) 7   c) 3   d) 5   e) 3   f) 2   g) 1   h) infinite   i) 0

3) All of the elements of the set $\{7,9,12\}$ are contained in the set $\{5,6,7,8,9,10,11,12\}$.

Therefore, the set $\{7,9,12\}$ is a subset of the set $\{5,6,7,8,9,10,11,12\}$.

4) Not all of the elements of the set $\{0,1,4,9,16\}$ are contained in the set $\{1,2,3,4,5,...\}$.

Specifically, 0 is an member of the first set, but not the second. Therefore, the set $\{0,1,4,9,16\}$ is not a subset of the set $\{1,2,3,4,5,...\}$.

5) a) iv   b) iii   c) i   d) ii

6) a) $\{1,2,3,4,5,6,7,8,9\}$ b) $\{-5,-3,-1,1,3,5\}$ c) $\{0,1,2,3,4,5,...\}$
    d) $\{...,-3,-2,-1,0,1,2,3,...\}$ e) $\{20,22,24,26,28,...\}$ f)
    $\{...,-15,-10,-5,0,5,10,15,...\}$

7) Answers will vary. For example, $\dfrac{1}{2}, \dfrac{2}{3}, 0.2$.

8) Answers will vary. For example, $3\dfrac{1}{4}, \dfrac{29}{8}, 3.5, 3.75$.

9) $\{\ \}, \{1\}, \{2\}, \{1,2\}$

10) $\{\ \}, \{-1\}, \{0\}, \{1\}, \{-1,0\}, \{-1,1\}, \{0,1\}, \{-1,0,1\}$

11) a) true   b) true   c) false   d) true   e) false   f) true   g) false

12) Jonah's claim is incorrect. A subset may have the same number of elements as its parent set. For example the parent set $\{1,2,3\}$ and its subset $\{1,2,3\}$ have the same number of members. (Note: A *proper subset* is a subset that is not equal to its parent set. A *proper subset* always has fewer elements than its parent set. No set is a *proper subset* of itself.)

13) a) Adding the element 0 to the set of natural numbers ($\mathbb{N}$) gives the set of whole numbers ($\mathbb{W}$). $\mathbb{N}$ is a subset of $\mathbb{W}$.   b) $\mathbb{W}$ is a subset of $\mathbb{Z}$.   c) $\mathbb{Z}$ is a subset of $\mathbb{Q}$.   d) The rational numbers ($\mathbb{Q}$) consist of all numbers of the form $\dfrac{a}{b}$, whereas the irrational numbers ($\mathbb{P}$) are all of the real numbers that cannot be expressed in that form. $\mathbb{Q}$ and $\mathbb{P}$ have no elements in common (they are *disjoint sets*).

14) Yes. Since C is a subset of B, all of the elements of C are contained in B. Since B is a subset of A, all of the elements of B are contained in A, and thus all of the elements of C are also contained in A. Therefore, C is a subset of A.

15) a) 10   b) 11   c) infinitely many   d) infinitely many   e) infinitely many

16) rational numbers, irrational numbers and real numbers

17) No. For example, there are no other even integers between 2 and 4.

18) a) 7   b) 0   c) 0   d) $\dfrac{1}{3}$ e) $-2\dfrac{2}{3}$ $\left(\text{or } -\dfrac{8}{3} \right)$ f) $1\dfrac{4}{9}$ $\left(\text{or } \dfrac{13}{9} \right)$

19) a) false   b) true   c) true   d) true   e) false

20) a) 8   b) 16   c) The total number of subsets is equal to 2^n.   d) 4096

21) a) The sequence starts with 1 and then each successive term is found by dividing the previous term by 2.

b) No. For example, 1 and $\dfrac{1}{2}$ are both members of the set, but there is no other member of the set that falls between these two values since they are all less than $\dfrac{1}{2}$.

c) The sequence has a limit of 0.

22) a) $\dfrac{25}{99}$ b) $\dfrac{3125}{999}$ c) 2