MCR 3U Functions and Relations Final Examination
(J a n u a r y)
PART A (21 marks)

Each correct answer has a value of one (1) mark.

1. Given $g(x) = 3 - 2x$, determine $g(4x)$. ________________

For the relation defined by $\frac{x^2}{49} + \frac{y^2}{16} = 1$:

(a) identify the type of conic ________________

(b) state the range ________________

(c) state the length of the major axis ________________

3. State all restrictions : $\frac{2}{x} \div \frac{x+2}{3}$ ________________

4. Evaluate: (express your answers as fractions)

(a) $16^{-\frac{3}{4}}$ ________________

(b) $3^{-1} + 3^0$ ________________

5. Describe the transformations required to obtain the graph of $y = -f(x+3)$ from a graph of a function defined by $y = f(x)$.

(a) ____

(b) ____

6. Given the recursion formula defined by $t_1 = -3,\ t_2 = 5,\ t_n = t_{n-2} - t_{n-1}$, determine t_3. ________________

State the conjugate of $-2 + 3i$. ________________

State the equation of one asymptote for the graph of $x^2 - y^2 = 1$. ________________

State the equation for the locus of points which are 5 units from $(-1, 0)$. ________________

3pie/4=135 degrees

10. State the exact value of $\cos\frac{3\pi}{4}$. ________________

11. θ is the measure of an angle with its terminal arm in the fourth quadrant such that $\cos\theta = 0.423$. Determine the value of θ to the nearest degree, $0° \le \theta \le 360°$. ________________

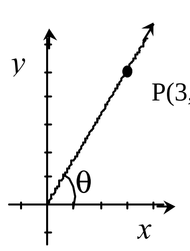
12. The first term of a sequence is –5 and the common ratio is 2.

(a) List the first three terms of this sequence. ________________

(b) State the general term. ________________

13. Simplify: $\dfrac{x^{\frac{3}{4}}}{x^{\frac{1}{4}}}$ ________________

14. Determine the value of $\sin\theta$. ________________



15. For what value of c does the equation of the function defined by $y = x^2 - 6x + c$ have only one x - intercept? ________________

16. Determine the number of zeroes of the function defined by $f(x) = -3(x-2)^2 - 5$. ________________

PART B (61 marks)

Each of the following questions requires a short answer completion in the space provided. Show all work. Mark values for each question appear in the left margin.

[3] Simplify: $(3-4i)^2 - i(i^3) + \frac{2}{i}$

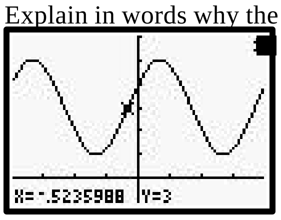
2pie=360degrees

2. A graphing calculator shows the following for a sine function with a period of 2π.

A student wrote the equation as $y = 2\sin\left(x - \frac{\pi}{6}\right) + 3$.

30 degrees

[1] (a) Explain in words why the student is incorrect.



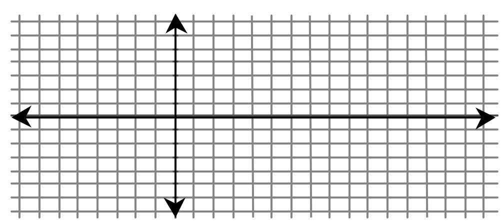
[1] (b) Write the correct equation.

3. Simplify. (It is not necessary to state restrictions)

[3] (a) $\frac{x}{3x-6} - \frac{2}{x^2-4}$

[3] (b) $\frac{2x+y}{2x^2} \div \frac{2x^2+3xy+y^2}{x^2+xy}$

[3] 4. Sketch $y = 3\cos\left(\frac{1}{2}x\right)+1$ for one cycle.



[3] 5. Prove the identity:

$$\frac{\tan^2\theta}{1+\tan^2\theta} = \sin^2\theta$$

6. Solve for θ:

2pie = 360 degrees

[3] (a) $2\sin\theta + 1 = 0, \ 0 \le \theta \le 2\pi$

[3] $4\tan^2\theta - 9 = 0, \ \ 0° \le \theta \le 360°$ (answer to the nearest degree)

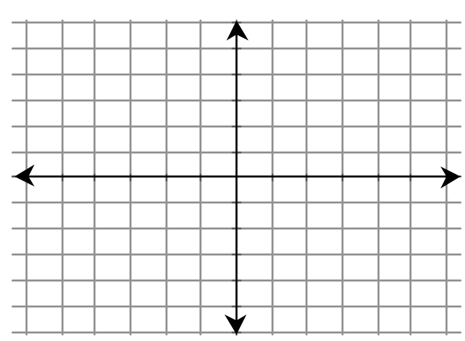
7. Given the relation f as defined by $y = \sqrt{x-2}$,

[2] (a) state the domain and the range of f.

[2] (b) sketch the graphs of f and f^{-1}.

[1] (c) does f represent a function? Explain your answer.

[2] (d) determine the expression for $f^{-1}(x)$.



[4] 8. Solve for x:

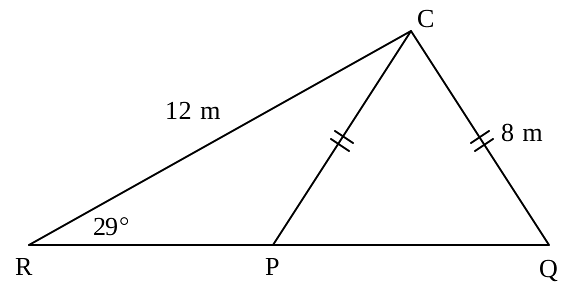
$$\left(2^x\right)^2 = 64\left(\frac{1}{32^x}\right)$$

9. A sporting goods store sells skates. During the first week, they sold 10 pairs of skates. In the second week they sold 14 pairs and in the third week they sold 18 pairs, and the pattern continues.

[1] (a) Identify the type of sequence. Explain.

[4] (b) How many weeks did it take to sell a total of 1450 pairs of skates? (Use the appropriate formula.)

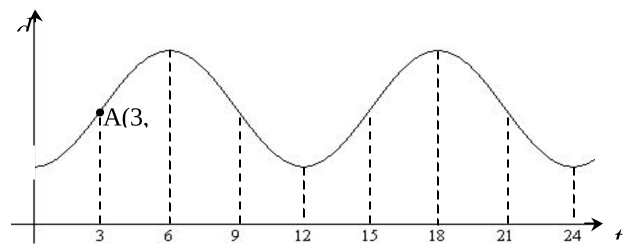
[4] 10. Determine the length of PQ, to the nearest metre.



11. Because of the tide, the depth of the water in a harbour is modelled by the equation $d = -3\cos\left(\frac{\pi}{6}t\right) + 6$, where d represents the depth of the water in metres and t represents the number of hours after midnight. (i.e. $t = 0$ means midnight, $t = 1$ means 1 A.M., and so on.)

pie/6=30 degrees

The graph of the relation is shown below:



[2] (a) What is the missing coordinate of point A? What do the coordinates of point A represent?

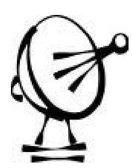
[1] (b) State the maximum depth of the water.

[2] (c) Surfing is allowed between 8 A.M. (08:00 hrs) and 7 P.M. (19:00 hrs), but only when the depth of the water is 6 m or more. For how many hours is surfing allowed in one day? Explain.

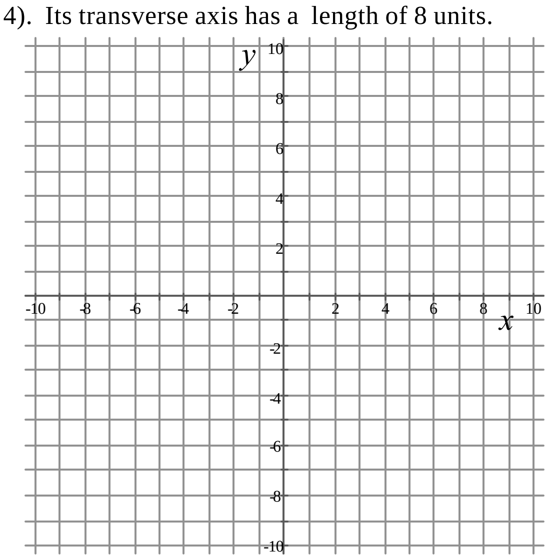
[3] (a) Express $9x^2 - 4y^2 - 36x - 8y = 4$ in standard form.

[2] (b) What are two advantages of writing the defining equation of a conic in standard form?

[3] The receiver of a parabolic satellite dish is at the focus. The focus is 72 cm from the vertex. If the dish is 240 cm in diameter, find the depth of the dish.



[5] A hyperbola has centre (2, -1) and one of its foci at (2, 4). Its transverse axis has a length of 8 units. Sketch the graph of the hyperbola.



OTTAWA-CARLETON DISTRICT SCHOOL BOARD

MCR 3U Functions and Relations Final Examination
(January)
PART A (21 marks)

Write only your answer for each of the following questions in the space provided. Each correct answer has a value of one (1) mark.

teacher use only

1. Given $g(x) = 3 - 2x$, determine $g(4x)$.

   $3-8x$

2. For the relation defined by $\frac{x^2}{49} + \frac{y^2}{16} = 1$:

   (a) identify the type of conic ellipse

   (b) state the range $\{y \mid y \in R, -4 \le y \le 4\}$ (accept inequality only)

   (c) state the length of the major axis 14

3. State all restrictions : $\frac{2}{x} \div \frac{x+2}{3}$ $x \neq 0, x \neq -2$

4. Evaluate: (express your answers as fractions)

   (a) $16^{-\frac{3}{4}}$ $\frac{1}{8}$

   (b) $3^{-1} + 3^0$ $\frac{4}{3}$

5. Describe the transformations required to obtain the graph of $y = -f(x+3)$ from a graph of a function defined by $y = f(x)$.

   (a) translation 3 units left

   (b) reflection in x-axis

6. Given the recursion formula defined by $t_1 = -3, t_2 = 5, t_n = t_{n-2} - t_{n-1}$, determine t_3. -8

7. ~~Determine the product~~ of $-2+3i$ ~~and its conjugate~~.
   State the conjugate
   $-2-3i$

8. State the equation of one asymptote for the graph of $x^2 - y^2 = 1$. $y=x$ or $y=-x$

9. State the equation for the locus of points which are 5 units from $(-1, 0)$. $(x+1)^2+y^2=25$

Content total for page = ________

Technical total for page = ________

teacher use only

10. State the exact value of $\cos\frac{3\pi}{4}$.

$-\frac{1}{\sqrt{2}}$

11. θ is the measure of an angle with its terminal arm in the fourth quadrant such that $\cos\theta = 0.423$. Determine the value of θ to the nearest degree, $0° \le \theta \le 360°$.

$295°$

12. The first term of a sequence is –5 and the common ratio is 2.

(a) List the first three terms of this sequence.

$-5, -10, -20$

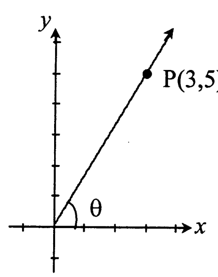
(b) State the general term.

$t_n = -5(2)^{n-1}$

13. Simplify: $\dfrac{x^{\frac{3}{4}}}{x^{\frac{1}{4}}}$

$x^{\frac{1}{2}}$

14. Determine the value of sin θ.



$\frac{5}{\sqrt{34}}$

accept $\sim 0.8575...$

15. For what value of c does the equation of the function defined by $y = x^2 - 6x + c$ have only one x - intercept?

9

16. Determine the number of zeroes of the function defined by $f(x) = -3(x-2)^2 - 5$.

0

Content total for page = ________

Technical total for page = ________

PART B (~~60~~ 61 marks)

Each of the following questions requires a short answer completion in the space provided. Show all work. Mark values for each question appear in the left margin. **teacher use only**

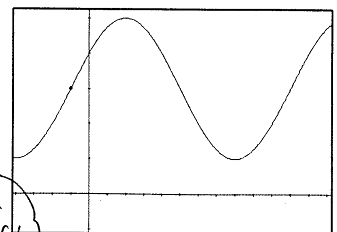
1. Simplify: $(3-4i)^2 - i(i^3) + \frac{2}{i}$

$= 9 - 24i - 16 - 1 - 2i$

[3] $= -8 - 26i$

✓ rewriting $\frac{2}{i}$ as $-2i$
✓ correctly simplifying $(3-4i)^2$ and collecting like terms
✓ knowing $i^2 = -1$

2. A graphing calculator shows the following for a sine function with a period of 2π. A student wrote the equation as $y = 2\sin\left(x - \frac{\pi}{6}\right) + 3$.



(a) Explain in words why the student is incorrect.

This graph is a shift left.

[1] ∴ The argument of sin should be $\left(x + \frac{\pi}{6}\right)$

✓ correct explanation citing shift left

(b) Write the correct equation.

[1] $y = 2\sin\left(x + \frac{\pi}{6}\right) + 3$

✓ correct equation

3. Simplify. (It is not necessary to state restrictions)

(a) $\frac{x}{3x-6} - \frac{2}{x^2-4}$

[3] $= \frac{x}{3(x-2)} - \frac{2}{(x-2)(x+2)}$

$= \frac{x^2+2x-6}{3(x-2)(x+2)}$

✓ factoring denominators correctly
✓ correct equivalent fractions
✓ simplifying

(b) $\frac{2x+y}{2x^2} \div \frac{2x^2+3xy+y^2}{x^2+xy}$

[3] $= \frac{2x+y}{2x^2} \cdot \frac{x(x+y)}{(2x+y)(x+y)}$

$= \frac{1}{2x}$

✓ multiplying by reciprocal
✓ factoring correctly
✓ reducing completely

Content total for page = ________

Technical total for page = ________

MCR 3U Functions and Relations Final Examination

teacher use only

PART B (~~60~~ 61 marks)

4. Sketch $y = 3\cos\left(\frac{1}{2}x\right) + 1$ for one cycle.

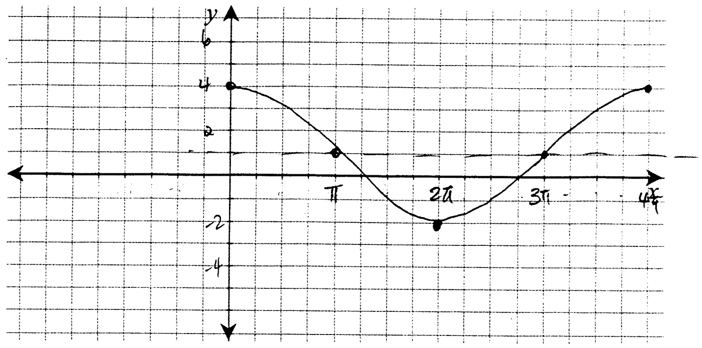
[3]

✓amplitude of 3

✓correct vertical translation

✓correct period

Ⓒ missing Scale



5. Prove the identity:

[3]

$$\frac{\tan^2\theta}{1+\tan^2\theta} = \sin^2\theta$$

$$LS = \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{1}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{1}{\frac{1}{\cos^2\theta}}$$

$= \sin^2\theta$

$= RS$

QED.

✓correctly working with $1+\tan^2\theta$ (finding common denominator)

✓knowing pyth. identity

✓knowing $\tan\theta = \frac{\sin\theta}{\cos\theta}$

6. Solve for θ:

(a) $2\sin\theta + 1 = 0,\ 0 \le \theta \le 2\pi$

[3]

$\sin\theta = -\frac{1}{2}$

related acute angle of θ is $\frac{\pi}{6}$

$\theta = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

accept equivalent decimal approxim.

✓correct acute angle

✓correct angle in QIII

✓correct angle in QIV

(b) $4\tan^2\theta - 9 = 0,\ 0° \le \theta \le 360°$ (answer to the nearest degree)

[3]

$\tan\theta = \pm\frac{3}{2}$

related acute angle of θ is approx. 56°

$\theta \doteq 56°, 124°, 236°, 304°$.

✓solving for $\tan\theta$

✓correct acute angle

✓all values of θ

N.B. if $\pm\frac{3}{2}$ ignored, award max. of 2 marks

Content total for page = ________

Technical total for page = ________

MCR 3U Functions and Relations Final Examination

teacher use only

PART B (~~60~~ 61 marks)

7. Given the relation f as defined by $y = \sqrt{x-2}$,

(a) state the domain and the range of f.

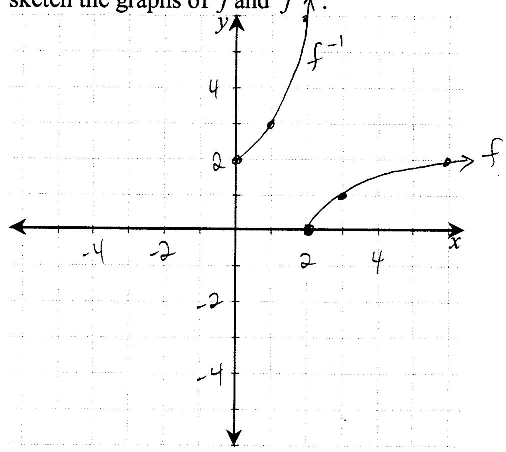
[2] $D = \{x \mid x \in R, x \geq 2\}$

$R = \{y \mid y \in R, y \geq 0\}$

✓ Domain
✓ Range
correct

(b) sketch the graphs of f and f^{-1}.

[2]



✓ correct f graph
✓ correctly graphing inverse of the graph of f shown

Ⓒ missing scale, missing labels

(c) does f represent a function? Explain your answer.

[1] Yes, since there is only one y-value for each x-value (or, since the graph of f passes the VLT)

✓ correct explanation

(d) determine $f^{-1}(x)$.

[2] $f^{-1}(x) = x^2 + 2, \ x \geq 0$

✓ correct equation
✓ correct restriction

8. Solve for x:

$$\left(2^x\right)^2 = 64\left(\frac{1}{32^x}\right)$$

[4] $2^{2x} = 2^6\left(2^{-5x}\right)$

$2^{2x} = 2^{6-5x}$

$2x = 6 - 5x$

$7x = 6$

$x = \frac{6}{7}$

✓ writing each power of 2
✓ using product or quotient rule correctly
✓ knowing to equate exponents
✓ solving for x

Content total for page = ________

Technical total for page = ________

PART B (~~60~~ 61 marks)

9. A sporting goods store sells skates. During the first week, they sold 10 pairs of skates. In the second week they sold 14 pairs and in the third week they sold 18 pairs, and the pattern continues.

(a) Identify the type of sequence. Explain.

[1]

Arithmetic, since there is a common difference between successive terms.

(b) How many weeks did it take to sell a total of 1450 pairs of skates? (Use the appropriate formula.)

[4]

$a = 10,\ d = 4$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$1450 = \frac{n}{2}[20 + 4(n-1)]$

$= 10n + 2n^2 - 2n$

$\rightarrow n^2 + 4n - 725 = 0$

$(n+29)(n-25) = 0$

$n = 25$ since $n > 0$

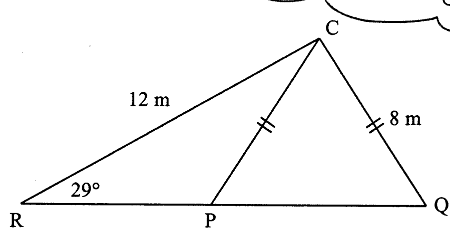
$\therefore$ It would take 25 weeks to sell 1450 pairs.

✓ correctly substituting into formula
✓ simplifying to standard quadr. eqⁿ
✓ factoring & solving
✓ appropriate conclusion

ⓒ not mentioning $n > 0$ when omitting -29

10. Determine the length of PQ, to the nearest metre.

[4]



$\frac{\sin\angle Q}{12} = \frac{\sin 29^\circ}{8}$

$\angle Q \doteq 46.7^\circ$

$\therefore \angle CPQ \doteq 46.7^\circ$

$\angle PCQ \doteq 86.7^\circ$

$PQ \doteq \sqrt{8^2 + 8^2 - 2(64)\cos 86.7^\circ}$

$\doteq 11$

✓ correct sin law relationship for $\angle Q$
✓ solving for $\angle Q$ and determining other angles in $\triangle CPQ$
or
✓ sine law and cos. law
✓ solving for PQ

Content total for page = ________

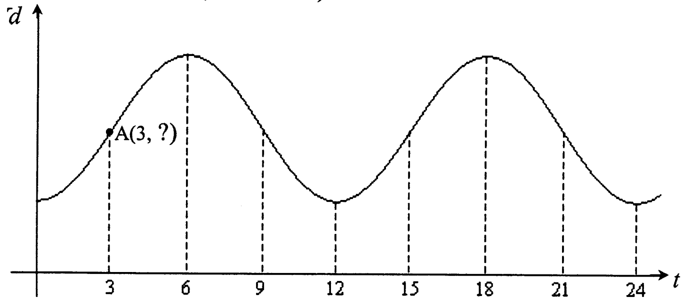
Technical total for page = ________

MCR 3U Functions and Relations Final Examination

teacher use only

PART B (~~60~~ 61 marks)

11. Because of the tide, the depth of the water in a harbour is modelled by the equation $d = -3\cos\left(\frac{\pi}{6}t\right) + 6$, where d represents the depth of the water in metres and t represents the number of hours after midnight. (i.e. $t = 0$ means midnight, $t = 1$ means 1 A.M., and so on.)



(a) What is the missing coordinate of point A? What do the coordinates of point A represent?

[2]

A(3,6) means that the (mean) depth of 6 m occurs at 3 AM

✓ d(3) correct
✓ explanation

(b) What is the maximum depth of the water?

[1]

9 m

✓ correct value

(c) Surfing is allowed between 8 A.M. and 7 P.M. when the depth of the water is 6 m or more. For how many hours is surfing allowed in one day?

[2]
~~[1]~~

In the 11 hour period, the depth is less than 6 m from 9 AM to 3 PM

∴ Surfing is allowed for 5 h

✓ correct value
✓ explanation

12. (a) Express $9x^2 - 4y^2 - 36x - 8y = 4$ in standard form.

[3]

$9(x^2 - 4x + 4) - 4(y^2 + 2y + 1) = 4 + 36 - 4$

$9(x-2)^2 - 4(y+1)^2 = 36$

$\frac{(x-2)^2}{4} - \frac{(y+1)^2}{9} = 1$

✓ correctly completing the square for x
✓ and y
✓ standard form

(b) What are two advantages of writing the defining equation of a conic in standard form?

[2]

- features of the graph of the relation, like the centre, intercepts, asymptotes are evident
- identify the type of conic more easily
- easier to graph

✓
✓ } any two advantages

Content total for page = ________

Technical total for page = ________

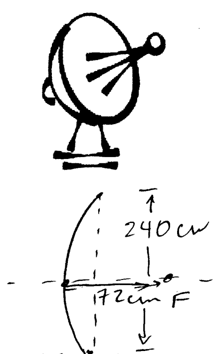
MCR 3U Functions and Relations Final Examination

teacher use only

PART B (~~60~~ 61 marks)

13. The receiver of a parabolic satellite dish is at the focus. The focus is 72 cm from the vertex. If the dish is 240 cm in diameter, find the depth of the dish.

[3]



equation of parabola

$y^2 = 4px$, $p = 72$

$y^2 = 288x$

if $y = 120$, $120^2 = 288x$

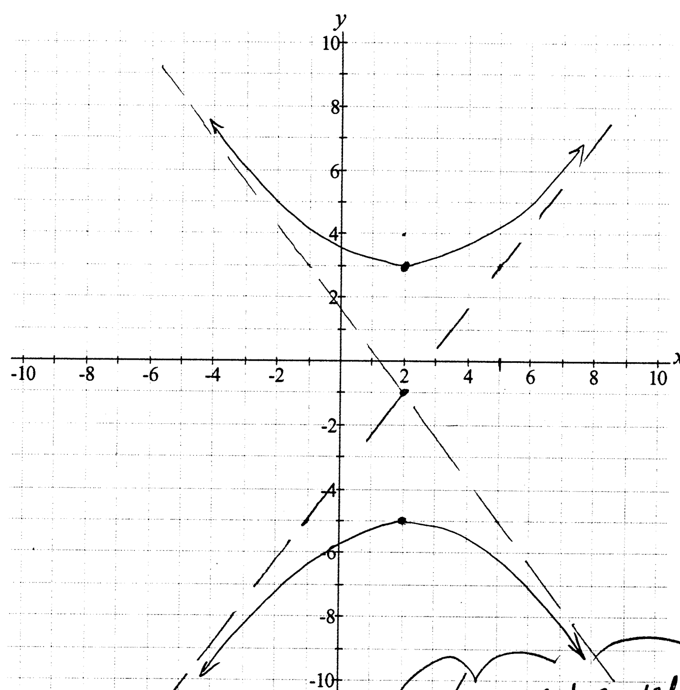
$x = 50$

$\therefore$ The depth is 50 cm

✓ knowing $p = 72$
✓ determining eqn of parabola
✓ solving for depth

14. A hyperbola has centre (2, -1) and one of its foci at (2, 4). Its transverse axis has a length of 8 units. Sketch the graph of the hyperbola.

[5]



$c = 5$

$a = 4$

$c^2 = a^2 + b^2$

$\therefore b = 3$

slope of asymptotes $= \pm \frac{4}{3}$

✓ correct c-value
✓ a-value
✓ b-value
✓ location of asymptotes
✓ sketch with appropriate shape

Content total for page = ________

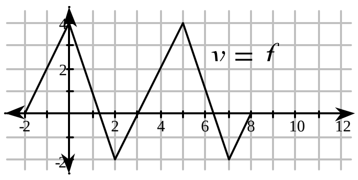
Technical total for page = ________

MCR 3U Functions & Relations Final Examination
(B a c k u p)
PART A (20 marks)

Write only your answer for each of the following questions in the space provided. Each correct answer has a value of one (1) mark.

1. If $f(x) = 5x^2 - 2$, determine $f(-3)$. ____________

2. For the given periodic relation, state:



(a) the period ____________

(b) the amplitude ____________

(c) the value of $f(11)$ assuming the relation continues in the same manner. ____________

3. Evaluate $8^{-\frac{5}{3}}$. (Express answer as a fraction) ____________

4. Given $y = 2\sqrt{x-5}$, state:

(a) the domain ____________

(b) the range ____________

Express $\sqrt{-25}$ in terms of i. ____________

Evaluate i^6. ____________

7. State the restrictions for $\frac{x-3}{x^2(x-3)}$. ____________

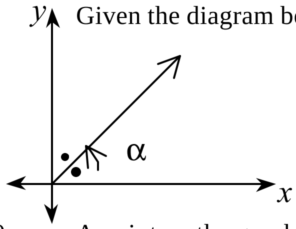
5pie/6 = 150 degrees

8. Given $\theta = \frac{5\pi}{6}$, state:

(a) the measure of θ in degrees ____________

(b) the exact value of $\cos\theta$. ____________

9. Given the diagram below, state the exact measure of α in radians. ____________



10. A point on the graph of $y = f(x)$ is $(8,-3)$. The coordinates of the corresponding image point

(a) on the graph of $y = 2f(x)$ are ____________

(b) on the graph of $y = f(x+2)$ are ____________

(c) on the graph of $y = f^{-1}(x)$ are ____________

11. Given the recursion formula defined by $t_1 = 5$, $t_n = 2t_{n-1} - 3$, determine t_2. ____________

Given the conic defined by $y^2 = -8x$, determine:

(a) the coordinates of the focus. ____________

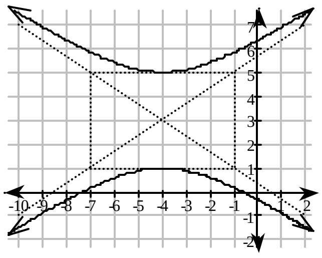
(b) the equation of the directrix. ____________

13. Simplify $a^{\frac{5}{4}} \cdot a^{\frac{3}{4}}$ ____________

PART B (67 marks)

Each of the following questions requires a short answer completion in the space provided. Show all work. Mark values for each question appear in the left margin.

[3] Find the defining equation of the conic whose graph is shown below. Express your answer in standard form.



[3] Simplify $\dfrac{5+3i}{4-i}$

3. $P(-2,-3)$ lies on the terminal arm of the angle in standard position with measure θ. Determine:

[2] (a) the exact value of $\sin\theta$.

[2] (b) the value of θ to the nearest degree, where $0^\circ \le \theta \le 360^\circ$.

[3] 4. Simplify completely: (It is not necessary to state restrictions.)

$$\frac{a}{a+3}+\frac{9a}{3a^2+8a-3}$$

[4] 5. Simplify and state the restrictions

$$\frac{2m+3}{2m-3}\div\frac{m+3}{9-4m^2}$$

[4] 6. An arrow is shot from the roof of a building. Its height above the ground is modelled by $h(t)=-5t^2+40t+20$, where h is the height in metres and t is the time elapsed in seconds, from the time the arrow was shot. For what length of time is the arrow more than 35 m above the ground? Express your answer to the nearest tenth of a second.

[3] 7. Prove the identity:

$$\tan\theta-\frac{1}{\tan\theta}=\frac{2\sin^2\theta-1}{\sin\theta\cos\theta}$$

8. Solve for θ:

```
2pie=360 degrees
```

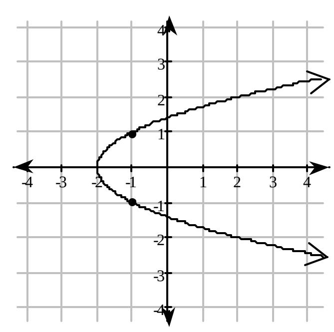
[2] (a) $\tan\theta-\sqrt{3}=0,\ 0\le\theta\le 2\pi$ (exact values)

[3] $3\cos^2\theta-7\cos\theta+2=0,\ 0\le\theta\le 2\pi$ (round answers correct to 2 decimal places)

9. If you were given a function in the form $y=f(x)$, explain how you would determine the defining equation of its inverse, namely $y=f^{-1}(x)$.

[2]

10. The graph of a parabolic relation is shown.



[1] (a) State the domain.

[1] (b) Graph the inverse on the same grid.

[1] (c) Consider the statement: "Since the given relation is not a function, then its inverse is not a function." Is this statement true? Explain your answer.

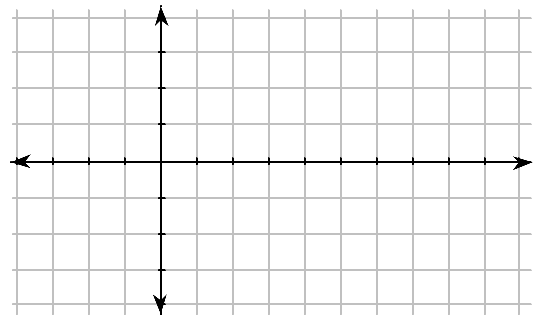
[3] 11. Solve for x:

$$27^{x-2}=\frac{1}{9^x}$$

pie/2=90 degrees

[4] 12. Sketch one cycle of the following trigonometric function:

$$y = -2\sin\left(3x + \frac{\pi}{2}\right)$$

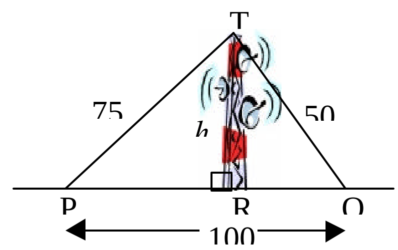


13. Given the series $800 + 400 + 200 + 100 + \ldots$, using the appropriate formulas,

[2] (a) determine t_{12} to 3 decimal places.

[2] (b) determine S_{12} to the nearest decimal place.

[4] 14. Two guy wires as shown in the diagram support a microwave tower. What is the height, h metres, of the tower, to the nearest metre?



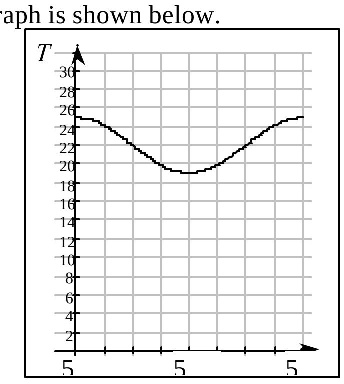
[3] 15. You have the opportunity to work between 1 and 50 hours during the March Break. You can choose the method of payment from the following:

Choice 1: You can be paid $15 per hour

Choice 2: You can be paid $1 for the first hour, $2 for the second hour, $3 for the third hour, and the pattern continues.

What are the advantages of each choice? Justify your answers.

16. The inside temperature of a building is modelled by $T(t) = 3\cos(0.262t) + 22$, where T is the temperature in °C and t is the number of hours elapsed since 5 A.M. The graph is shown below.



[2] (a) Using an appropriate calculation, explain why the coefficient of t in the equation is 0.262.

[2] (b) In another building, the temperature fluctuates in a similar manner except that the maximum temperature is 27°C and the minimum temperature is 23°C. Determine the defining equation that models the temperature in this other building.

17. A radar screen shows the activity within a circular region of radius 60 km.

[1] (a) Assuming the centre of the screen is (0, 0), write the equation that represents this circle.

[4] (b) A small aircraft flies on a path given by the equation $x + 2y = 140$. Is this small aircraft detected on the radar screen? Explain your answer algebraically.

Given the conic defined by $25x^2 + 9y^2 - 100x + 18y - 116 = 0$, determine:

[2] (a) the coordinates of the centre

[3] (b) the coordinates of the foci.

MCR 3U Functions & Relations Final Examination

(Backup)

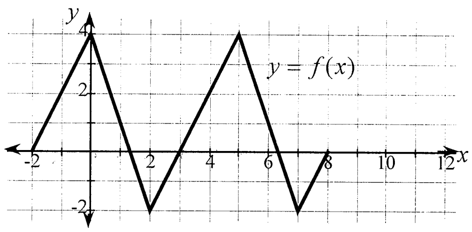
PART A (20 marks)

Write only your answer for each of the following questions in the space provided. Each correct answer has a value of one (1) mark.

teacher use only

1. If $f(x) = 5x^2 - 2$, determine $f(-3)$. 43

2. For the given periodic relation, state:



(a) the period 5

(b) the amplitude 3

(c) the value of $f(11)$ assuming the relation continues in the same manner. 1

3. Evaluate $8^{-\frac{5}{3}}$. (Express answer as a fraction) $\frac{1}{32}$

4. Given $y = 2\sqrt{x-5}$, state:

(a) the domain $\{x \mid x \in R, x \geq 5\}$

(b) the range $\{y \mid y \in R, y \geq 0\}$

∴ accept inequalities only

5. Express $\sqrt{-25}$ in terms of i. $5i$

6. Evaluate i^6. -1

7. State the restrictions for $\frac{x-3}{x^2(x-3)}$. $x \neq 0, 3$

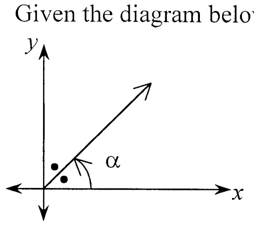
∴ $\boxed{0,3 \text{ only}}$

8. Given $\theta = \frac{5\pi}{6}$, state:

(a) the measure of θ in degrees $150°$

(b) the exact value of $\cos\theta$. $-\frac{\sqrt{3}}{2}$

9. Given the diagram below, state the exact measure of α in radians.



$\frac{\pi}{4}$

∴ do not accept 45°

Content total for page = ________

Write only your answer for each of the following questions in the space provided. Each correct answer has a value of one (1) mark.

teacher use only

10. A point on the graph of $y = f(x)$ is $(8,-3)$. The coordinates of the corresponding image point

(a) on the graph of $y = 2f(x)$ are (8, -6)

(b) on the graph of $y = f(x+2)$ are (6, -3)

(c) on the graph of $y = f^{-1}(x)$ are (-3, 8)

11. Given the recursion formula defined by $t_1 = 5$, $t_n = 2t_{n-1} - 3$, determine t_2.

7

12. Given the conic defined by $y^2 = -8x$, determine:

(a) the coordinates of the focus. (-2, 0)

(b) the equation of the directrix. $x = 2$

• no mark for "2"

13. Simplify $a^{\frac{5}{4}} \cdot a^{\frac{3}{4}}$

a^2 ⓒ $\boxed{a^{8/4}}$

Content total for page = ________

PART B (67 marks)

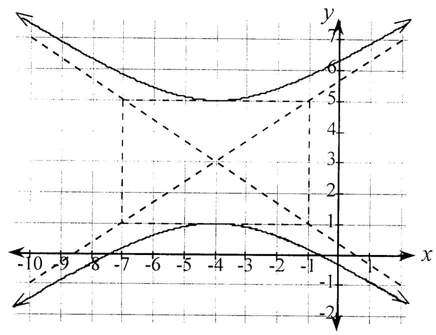
Each of the following questions requires a short answer completion in the space provided. Show all work. Mark values for each question appear in the left margin.

teacher use only

1. Find the defining equation of the conic whose graph is shown below. Express your answer in standard form.

[3]

$$\frac{(x+4)^2}{9} - \frac{(y-3)^2}{4} = -1$$



✓correct numerators
✓correct denominators
✓correct form.

2. Simplify $\frac{5+3i}{4-i}$

[3]

$$\frac{5+3i}{4-i} = \frac{5+3i}{4-i} \cdot \frac{4+i}{4+i}$$

$$= \frac{20+17i-3}{17}$$

$$= \frac{17+17i}{17}$$

$$= 1+i$$

✓multiplying by correct conjugate
✓expanding each binomial correctly.
✓knowing $i^2 = -1$

© $\boxed{\text{not reducing to } 1+i}$

3. P(−2,−3) lies on the terminal arm of the angle in standard position with measure θ. Determine:

(a) the exact value of sin θ.

[2]

$r = \sqrt{13}$

$\sin\theta = -\frac{3}{\sqrt{13}}$

✓correct r value
✓knowing $\sin\theta = \frac{y}{r}$

(b) the value of θ to the nearest degree, where $0° \le \theta \le 360°$.

[2]

the related acute angle of $\theta = \sin^{-1}\left(\frac{3}{\sqrt{13}}\right)$
$\doteq 56°$

P is in QIII $\therefore \theta \doteq 180° + 56°$
$= 236°$

© $\boxed{\doteq \text{ where nec.}}$

✓correct related acute angle
✓correct θ value

4. Simplify completely: (It is not necessary to state restrictions.)

$$\frac{a}{a+3} + \frac{9a}{3a^2+8a-3}$$

[3]

$$= \frac{a}{a+3} + \frac{9a}{(a+3)(3a-1)}$$

$$= \frac{3a^2-a+9a}{(a+3)(3a-1)}$$

$$= \frac{3a^2+8a}{(a+3)(3a-1)}$$

✓factoring correctly
✓writing correct equivalent fractions with correct LCD
✓simplifying correctly.

Content total for page = ________

teacher use only

PART B (67 marks)

5. Simplify and state the restrictions

$$\frac{2m+3}{2m-3} \div \frac{m+3}{9-4m^2}$$

[4]

$$= \frac{2m+3}{2m-3} \cdot \frac{(3-2m)(3+2m)}{m+3}$$

$$= \frac{-(2m+3)^2}{m+3}$$

$m \neq \pm \frac{3}{2}, -3$

✓ factoring correctly
✓ • by reciprocal
✓ reducing correctly
✓ correct restrictions

6. An arrow is shot from the roof of a building. Its height above the ground is modelled by $h(t) = -5t^2 + 40t + 20$, where h is the height in metres and t is the time elapsed in seconds, from the time the arrow was shot. For what length of time is the arrow more than 35 m above the ground? Express your answer to the nearest tenth of a second.

The arrow is 35 m above the ground when

[4]

$$-5t^2 + 40t + 20 = 35$$
$$-5t^2 + 40t - 15 = 0$$
$$t^2 - 8t + 3 = 0$$
$$t = \frac{8 \pm \sqrt{64-12}}{2}$$
$$t \doteq 0.4 \text{ or } t \doteq 7.6$$

∴ The arrow is above 35 m for approximately 7.2 s

✓ correct equation to apply Qudd. formula to.
✓ substituting correctly in correct formula
✓ two correct values
✓ knowing to find the difference

Ⓒ incorrect use of units
Ⓒ missing statements

7. Prove the identity:

$$\tan\theta - \frac{1}{\tan\theta} = \frac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$$

[3]

$$LS = \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{\sin^2\theta - (1-\sin^2\theta)}{\sin\theta\cos\theta}$$
$$= \frac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$$
$= R.S.$ QED.

✓ knowing $\tan\theta = \frac{\sin\theta}{\cos\theta}$
✓ knowing correct use of Pythagorean Identity
✓ correct algebraic simplification

8. Solve for θ:

(a) $\tan\theta - \sqrt{3} = 0$, $0 \le \theta \le 2\pi$ (exact values)

[2]

$\tan\theta = \sqrt{3}$

the related acute angle of θ is $\frac{\pi}{3}$

$\tan\theta > 0$ in QI, QIII
$\therefore \theta = \frac{\pi}{3}, \frac{4\pi}{3}$

✓ correct related acute angle
✓ correct θ values (both)

(b) $3\cos^2\theta - 7\cos\theta + 2 = 0$, $0 \le \theta \le 2\pi$ (round answers ~~correct to 2 decimal~~ places)

[3]

$(3\cos\theta - 1)(\cos\theta - 2) = 0$

$\cos\theta = \frac{1}{3}$ or $\cos\theta = 2$

but $-1 \le \cos\theta \le 1$

$\therefore \cos\theta = \frac{1}{3}$

related acute angle of θ $= \cos^{-1}\left(\frac{1}{3}\right)$
$\doteq 1.23$

$\cos\theta > 0$ in QI, QIV
$\therefore \theta \doteq 1.23$
or $\theta \doteq 5.05$

✓ factoring correctly
✓ knowing to eliminate 2
✓ correct related angle
✓ correct QIV angle

Content total for page = 58

Ⓒ angle measured in degrees

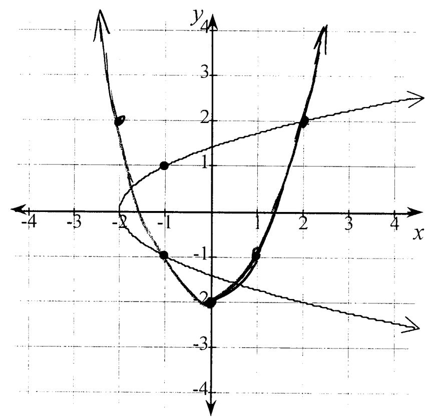
PART B (67 marks)

9. If you were given a function in the form $y = f(x)$, explain how you would determine the defining equation of its inverse, namely $y = f^{-1}(x)$.

[2] • Switch x and y in the equation and solve for y.

√ switch √ solve

10. The graph of a parabolic relation is shown.



(a) State the domain.

[1] $\{x \mid x \in R,\ x \geq -2\}$

√ correct domain

[1]
(b) Graph the inverse on the same grid.

√ correct graph

(c) Consider the statement: "Since the given relation is not a function, then its inverse is not a function." Is this statement true? Explain your answer.

[1]

No. The inverse passes the vertical line test and the original function doesn't
∴ the original is not a function but the inverse is.

√ answer of "no", with valid explanation

11. Solve for x:

$$27^{x-2} = \frac{1}{9^x}$$

[3]

$(3^3)^{x-2} = (3^{-2})^x$

$3^{3x-6} = 3^{-2x}$

$3x - 6 = -2x$

$5x = 6$

$x = \frac{6}{5}$

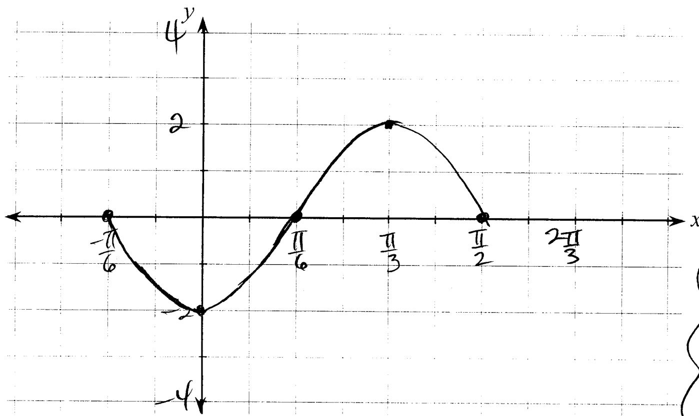
√ changing to base 3 on each side
√ equating exponents
√ solving for x correctly

teacher use only

PART B (67 marks)

12. Sketch one cycle of the following trigonometric function: [4]

$y = -2\sin\left(3x + \frac{\pi}{2}\right) = -2\sin\left[3\left(x + \frac{\pi}{6}\right)\right]$



period $= \frac{2\pi}{3}$
p.s. $= -\frac{\pi}{6}$
ampl. $= 2$
refl in x-axis

✓ correct period on graph
✓ correct amplitude on graph
✓ refl in x-axis
✓ correct shift

C error on scale

−1 if cos graph drawn instead of sin graph

13. Given the series $800 + 400 + 200 + 100 + \ldots$, using the appropriate formulas,

(a) determine t_{12} to 3 decimal places. [2]

geometric series
$a = 800 \quad t_n = ar^{n-1}$
$r = \frac{1}{2} \quad t_{12} = 800\left(\frac{1}{2}\right)^{11}$
$\doteq 0.391$

✓ t_n formula correct
✓ t_{12} value correct

(b) determine S_{12} to the nearest decimal place. [2]

$S_n = \frac{a(r^n - 1)}{r - 1}$

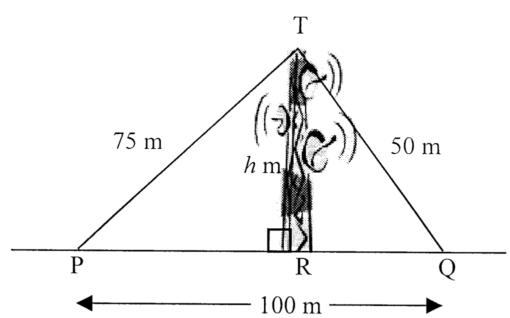
$S_{12} = \frac{800\left[\left(\frac{1}{2}\right)^{12} - 1\right]}{-\frac{1}{2}}$

$\doteq 1599.6$

✓ S_{12} equation correct
✓ S_{12} value

14. Two guy wires as shown in the diagram support a microwave tower. What is the height, h metres, of the tower, to the nearest metre? [4]

$\cos\angle P = \frac{75^2 + 100^2 - 50^2}{2(75)(100)}$



$\angle P \doteq 28.96°$

$\sin\angle P = \frac{h}{75}$

$h \doteq 75 \sin 28.96°$
$\doteq 36.3$

∴ The height of the tower is approximately 36 m tall.

✓ correct application of cos law
✓ determining $\angle P$ (or $\angle Q$) correctly
✓ sin. relationship for h
✓ solving for h correctly

C · consider misuse of units
· missing statement(s)

Content total for page = ________

teacher use only

PART B (66 marks)

15. You will be scheduled to work 40 hours during the March Break. You can choose the method of payment from the following:

Choice 1: You can be paid \$15 per hour

Choice 2: You can be paid \$1 for the first hour, \$2 for the second hour, \$3 for the third hour, and the pattern continues.

What is your choice? Justify your answer.

[3]

With choice ①, total earnings $= \$15(40)$
$= \$600$

with choice ②, the total earnings is the sum of the arithmetic series $S_n = \frac{n}{2}(a + t_n)$
with $n = 40$, $a = 1$, $t_n = 40$.
$\therefore S_{40} = \$20(41)$
$= \$820$

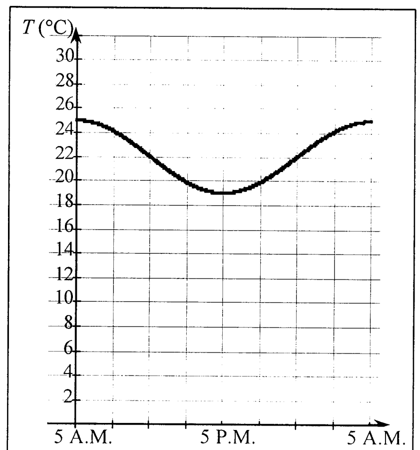
$\therefore$ Total earnings of ② are higher $\therefore$ I would choose ②

✓ correct total earnings of ①
✓ correct formula for ②
✓ correct total earnings from ②

16. The inside temperature of a building is modelled by $T(t) = 3\cos(0.262t) + 22$, where T is the temperature in °C and t is the number of hours elapsed since 5 A.M. The graph is shown below.

(a) Using an appropriate calculation, explain why the coefficient of t in the equation is 0.262.

[2]



$k = 0.262$

period $= \frac{2\pi}{k}$

$= \frac{2\pi}{0.262}$

$\doteq 24$

$\therefore$ a k-value of 0.262 allows the period to be 24 hrs (one day)

✓ knowing period $= \frac{2\pi}{k}$
✓ using the fact of 24 h period in explanation

(b) In another building, the temperature fluctuates in a similar manner except that the maximum temperature is 27°C and the minimum temperature is 23°C. Determine the defining equation that models the temperature in this other building.

[2]

amplitude $= \frac{27-23}{2}$
$= 2$

vertical shift $= \frac{27+23}{2}$
$= 25$

✓ correct amplitude
✓ correct vertical shift

$\therefore T(t) = 2\cos(0.262t) + 25$

Content total for page = ________

teacher use only

PART B (67 marks)

17. A radar screen shows the activity within a circular region of radius 60 km.

(a) Assuming the centre of the screen is (0, 0), write the equation that represents this circle.

[1] $x^2+y^2=3600$

✓ correct equation

(b) A small aircraft flies on a path given by the equation $x+2y=140$. Is this small aircraft detected on the radar screen? Explain your answer algebraically.

The plane is detectable if the line and circle intersect

For the intersection pts, if they exist,

[4]

$(140-2y)^2+y^2=3600$

$19600-560y+4y^2+y^2=3600$

$5y^2-560y+16000=0$

$y^2-112y+3200=0$

Check the discriminant for number of solutions: $D=112^2-4(3200)$ $=-256$

Since $D<0$, the line and circle do not meet

∴ the plane is not detected on the screen

✓ knowing to look for an intersection pt by creating a system or statement
✓ substituting and simplifying to std quad. eqn
✓ using a valid method of justification
✓ correct conclusion based on work done

18. Given the conic defined by $25x^2+9y^2-100x+18y-116=0$, determine:

(a) the coordinates of the centre

[2]

$25(x^2-4x+4)+9(y^2+2y+1)=116+100+9$

$25(x-2)^2+9(y+1)^2=225$

∴ The centre is $(2,-1)$

✓ correctly completing both squares
✓ stating centre based on work done

(b) the coordinates of the foci.

$$\frac{(x-2)^2}{9}+\frac{(y+1)^2}{25}=1$$

[3]

$a=5$, $b=3$ ∴ $c=4$

∴ The foci are $(2,3)$ and $(2,-5)$

Using work done in (a),
✓ determining "a" an "b"
✓ determining "c"
✓ stating foci

Content total for page = ________