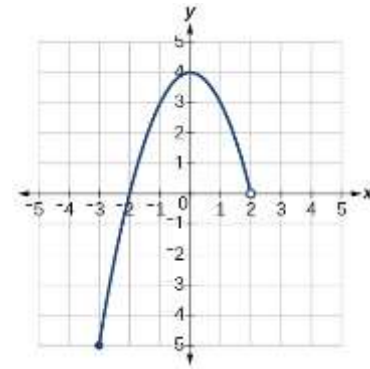# Unit 1: Functions

1. State the domain and range.

Is this a function?

2. Given that $f(x) = x^2 + 7$ and $g(x) = -3x + 1$:

a) Evaluate $f(-2)$.   b) Simplify $4g(b + c)$.

c) Simplify $f(g(x))$.   d) Solve for x if $f(x) = 23$.

3. For the quadratic function $f(x) = -5x^2 + 15x - 9$, determine the min/max value and when it occurs, by:

a) Completing the square   b) Partial factoring

4. Simplify.

a) $\sqrt{48} - \sqrt{27} + \sqrt{12}$ b) $5\sqrt{3} \times 3\sqrt{2}$

c) $\dfrac{15\sqrt{48}}{5\sqrt{3}}$ d) $\dfrac{3}{\sqrt{3} - 4}$

5. Factor fully: $2x^2 + 6x^3 + 5x^7 + 15x^8$.

6. Solve:

a) $6x^2 + 5x = 4$

b) $4x^2 - 10x + 5 = 0$

7. Algebraically determine the equation of a parabola which passes through the point $(1, -1)$ with roots at $2 \pm \sqrt{3}$.

8. The sum of the squares of two consecutive odd numbers is 394. Algebraically determine the numbers.

9. Solve the following linear quadratic system:

① $2y + 6 = x$

② $y^2 - 9 = 0$

Unit 2

1. Simplify. State any restrictions.

a) $\dfrac{x^2-16}{x^2-x-12}$

b) $\dfrac{2x^2-x-1}{3x^2+x-2} \div \dfrac{2x^2-3x-2}{3x^2-11x+6}$

c) $\dfrac{x+1}{3x^2+4x+1} + \dfrac{2x-1}{3x^2-5x-2}$

2. Describe the transformations, in order, of the following function from the graph of $f(x)$:

$$g(x) = -2f\left(\frac{1}{3}(x+3)\right) - 6$$

3. Given $f(x) = x^2 + 6x$,
a) Write equations for $-f(x)$ and $f(-x)$.
b) Determine any points that are invariant.

4. Sketch the following graphs showing key points:

a) $y = \sqrt{-2x + 6}$

b) $y = -\frac{1}{2}|x| + 3$

c) $f(x) = \frac{4}{x-2} - 3$

d) $f(x) = \left(\frac{1}{2}x\right)^3$

5. Algebraically determine the inverse of the function $f(x) = 2(x + 4)^2 - 1$. Is the inverse a function?

Unit 3

1. Simplify. No decimals may be used.

a) $\left(\dfrac{4x^{-3}y^4}{8x^2y^{-2}}\right)^{-2}$

b) $\left(\dfrac{27}{125}\right)^{-\frac{2}{3}}$

c) $\dfrac{\left(p^{\frac{-3}{4}}q^3\right)^{\frac{1}{3}}}{(p^{-2}q^4)(p^2q^4)^{\frac{1}{2}}}$

d) $\left(\sqrt[4]{\sqrt[3]{\sqrt{64}}}\right)^8$

2. Given the function $y = -3(2)^{x-2} + 1$,

a) Describe the transformations.

b) Graph the function.

c) State the y-intercept, the domain, the range and

the equation of the asymptote.

3. Solve the following exponential equations:

a) $5(4^x) = 10$

b) $16^{2p+1} = 8^{4p+5}$

c) $3^{3x-1} = \dfrac{1}{81}$

d) $2^{x+5} + 2^x = 1056$

4. The half-life of Vanadium-48 is 32 hours. How long will it take an initial amount of 368 g to decay to 23 g?

Unit 4

1. The point $(-20, -21)$ is on the terminal arm of an angle θ in standard position. Find $\sin \theta$ and $\cos \theta$.

2. Evaluate exactly:

a) $\sin 330°$ b) $\sec 120°$ c) $\tan 270°$

3. Determine all values of θ, where $0° \leq \theta \leq 360°$.

a) $\cos \theta = -\dfrac{1}{\sqrt{2}}$ b) $\sin \theta = -0.1573$

c) $\cot \theta = \dfrac{1}{\sqrt{3}}$ d) $\csc \theta = -1$

4. Two ships left Port Hope on Lake Ontario at the same time. One travelled at 12 km/h on a course of 235°. The other travelled at 15 km/h on a course of 105°. How far apart were the ships after four hours, to the nearest kilometer?

5. Solve ΔKLM, given $< K = 20°$, $k = 2\ cm$ and $l = 5\ cm$.

6. Prove the following identites:

a) $\dfrac{1}{sinx} - sinx = \dfrac{cosx}{tanx}$
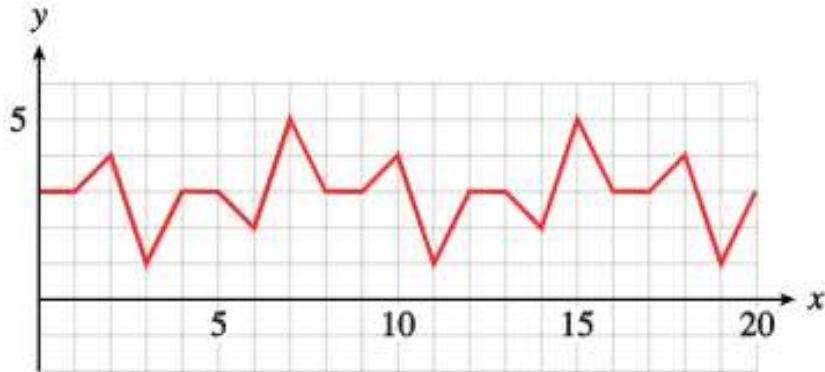
b) $\dfrac{1-tan^2x}{1+tan^2x} = cos^2x - sin^2x$

c) $2sin^4x - 3sin^2x + 1 = cos^2x(1 - 2sin^2x)$

Unit 5

1. For the periodic function shown, state the period, the amplitude and the equation of the midline.

2. Given $y = 3 \sin 3(\theta + 45°) + 1$, state the amplitude, period and phase shift. Graph one cycle of the function.

3. Write the equation of the cosine function that has an amplitude of 3, a period of 720°, a phase shift right 30° and a vertical translation down 2 units.

4. A Ferris wheel has a diameter of 20 m and is 4 m above ground level at its lowest point. The Ferris wheel completes one revolution in 120 seconds. Assuming that a rider enters a car from a platform that is located at the bottom of the Ferris wheel,

a) Model the rider's height above the ground versus time, using a transformed sine function.

b) Repeat part a) with a transformed cosine function.

Unit 6

1. Determine the simplified formula for the nth term and the indicated term for each sequence:

a) $-4, 3, 10, \ldots\,;\, t_{18}$

b) $1, -3, 9, \ldots\,;\, t_7$

2. Algebraically determine the number of terms for each sequence:

a) $19, 11, 3, \ldots, -229$

b) $27, 9, 3, \ldots, \dfrac{1}{2187}$

3. Write the first 5 terms of the sequence given the following recursion formula:

$$t_1 = 3;\ t_2 = 4;\ t_n = t_{n-1} + t_{n-2}$$

4. Find the sum of each series.

a) $1 + \dfrac{5}{4} + \dfrac{3}{2} + \cdots + 20$

b) $3645 - 1215 + 405 - \cdots + 5$

5. The Women's World Cup of Soccer tournament was first held in 1991. The next two tournaments were held in 1995 and 1999. Assuming that this pattern continues to repeat, algebraically determine the year of the 35th tournament.

6. One day you saw an awesome video on YouTube. At 1 pm, you shared a video link with 5 unique people. At 2 pm, each of your friends shared it with 5 unique people. At 3 pm, each of their friends shared it with 5 unique people. If this pattern keeps repeating, algebraically determine how many unique people have received the link by 11 pm.

7. Determine the missing portions of this section of Pascal's Triangle:

$$\boxed{120}\ \boxed{210}$$

$$\boxed{462}$$

$$\boxed{495}$$
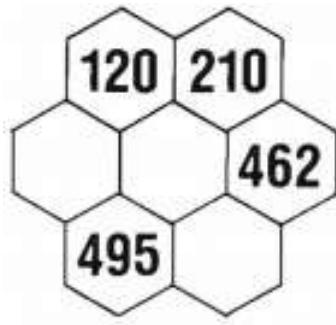
8. How many downward paths can be taken to spell MATHEMATICS?

M

A   A

T   T   T

H   H   H   H

E   E   E   E   E

M   M   M   M   M   M

A   A   A   A   A

T   T   T   T

I   I   I

C   C

S

9. Expand and simplify $\left(a - \dfrac{b}{2}\right)^5$.

Unit 7

1. Kadeem invested in a GIC that paid 3.25% simple interest. In 36 months, his investment grew to $485. How much did he invest originally?

2. Determine the amount of a $2200 investment, compounded monthly for 5 years at 12% per annum.

3. Faris needs $5000 for university in 3 years. His parents invest some money in an account paying interest at a rate of 7.1% per annum, compounded quarterly. How much should they invest now to have $5000 in 3 years?

4. Marianna deposited $200 into her bank account at the end of each month for 8 months. The account pays 2.9% per annum, compounded monthly. How much is in her account at the end of the 8 months?

5. Mrs. Stewart bought a new car. She financed

$13 500 at 3.9% compounded monthly and chose to make monthly payments for 4 years.

a) What amount does Mrs. Stewart pay per month?

b) How much interest did Mrs. Stewart pay to the dealership in order to finance her new car?