

7.5 Problem Solving - Mortgages

TVM Solver for Mortgage Calculations

- N = Total number of payments (# of payments X # of years)
- I% = Annual interest rate as a percent
- PV = Present value, or Amount of the mortgage
- PMT = The payment amount (a negative value (-) for mortgages)
- FV = Future value ("0" for paid-off mortgage, otherwise balance of the mortgage)
- P/Y = Number of payments per year
- C/Y = Number of compound periods per year
- PMT: = END for mortgages

*→ N is always # pmts  
Except  $A = P(1+i)^n$*

Important Notes:

- In Canada, mortgage interest is always compounded semi-annually, but in the U.S., mortgage interest is compounded monthly. Payments may be made at a different time i.e. monthly or bi weekly, so P/Y and C/Y do not need to match.
- Always input C/Y = 2 after P/Y, or the calculator automatically resets C/Y to match the P/Y.
- Cash outflows, like Mortgage Payments, are negative.
- Cash inflows, like the Mortgage Amount, are positive.
- The most common term for mortgages is a five year term. After 5 years you must renew the mortgage, which means taking out a new mortgage at current interest rates for the balance owing after 5 years.

Mortgage Vocabulary

|                   |                     |                     |                     |
|-------------------|---------------------|---------------------|---------------------|
| mortgage          | mortgage payment    | mortgagor           | mortgagee           |
| mortgage broker   | principal           | equity              | collateral          |
| down payment      | payment frequency   | accelerated payment | amortization period |
| fixed rate        | variable rate       | CMHC                | mortgage insurance  |
| land transfer tax | home inspection fee | closing costs       | length of term      |

- Mortgage: A special loan that has a "lien" as security.
- Mortgagor: The borrower of the money.
- Mortgagee: The lender of the money.
- Principal: The amount of money borrowed.
- Equity: The home's current market value.
- Collateral: Something forfeited if you default on a loan.
- Down Payment: Reduction from principal.
- Payment Frequency: How often you pay.
- Accelerated Payment: Monthly payment divided by 4 weeks - squeezes in an extra payment per year which means less interest paid!
- Amortization Period: Amount of time to repay a mortgage in full.
- Fixed Rate: Interest rate is locked in for a certain amount of time.
- Variable Rate: Interest rate fluctuates with the market.
- CMHC: Canadian Mortgage and Housing Corporation - A government agency that manages the rules of mortgages.
- Length of term: The amount of time you commit to a rate and lender.

Ex. 1 You have a \$173,500 mortgage, with monthly payments, at 3.2%/a over 25 years.

a) Calculate the monthly payments.

$$\begin{aligned}
 N &= 25 \times 12 \\
 I\% &= 3.2 \\
 PV &= 173500 \\
 PMT &= \boxed{\text{PMT}} \rightarrow -838.99 \\
 FV &= 0 \\
 P/Y &= 12 \\
 C/Y &= 2 \text{ *ALWAYS*} \\
 PMT &: \text{END BEGIN}
 \end{aligned}$$

b) How much money have you paid over the first 5 years?

$$\begin{aligned}
 \text{Amt paid} &= 12 \times 5 \times 838.99 \\
 &= 50339.40
 \end{aligned}$$

(This is Principal + Interest)

c) How much of the money paid was from the principal?

$$\begin{aligned}
 N &= 5 \times 12 \\
 I\% &= 3.2 \\
 PV &= 173500 \\
 PMT &= -838.99 \\
 FV &= \boxed{\text{FV}} \rightarrow -148863.83 \\
 P/Y &= 12 \\
 C/Y &= 2 \\
 PMT &: \text{END BEGIN}
 \end{aligned}$$

d) How much of the money paid was interest?

$$\begin{aligned}
 \text{Total pd} &: \$50339.40 \\
 \text{Total principal} &: \$24636.17
 \end{aligned}$$

$$\begin{aligned}
 \therefore I &= 50339.4 - 24636.17 \\
 &= \$25703.23
 \end{aligned}$$

$\therefore$  You paid \$25703.23

$$\begin{aligned}
 \therefore \text{Principal paid} &= 173500 - 148863.83 \\
 &= \$24636.17
 \end{aligned}$$

e) How much money have you paid over the 25 years?

$$\begin{aligned}
 \text{Total} &= 838.99 \times 25 \times 12 \\
 &= \$251697
 \end{aligned}$$

$\therefore$  Total paid is \$251697

f) How much interest will you pay over 25 years?

$$\begin{aligned}
 \text{Interest} &= 251697 - 173500 \\
 &= 78197
 \end{aligned}$$

$\therefore$  Total interest \$78197

g) From your answers, do you pay off more interest or more principal in the first 5 years of your mortgage? Last 5 years?

↑  
More interest  
Less principal

↑  
Less interest  
More principal

# 7.5 Mortgage Problem Solving.notebook

June 06, 2023

Ex. 2 Given an interest rate of 5% for a mortgage of \$250 000, determine your monthly payments and compare the total amount of interest paid if you amortize the mortgage over 20 years and over 25 years. Discuss the pros and cons between both options.

20 years

25 years

$$\begin{array}{l}
 N = 20 \times 12 \\
 I\% = 5 \\
 PV = 250000 \\
 PMT = \boxed{\rightarrow} -1642.81 \\
 FV = 0 \\
 P/Y = 12 \\
 C/Y = 2 \\
 PMT: \text{END} \text{ BEGIN}
 \end{array}$$

$$\begin{array}{l}
 N = 25 \times 12 \\
 I\% = 5 \\
 PV = 250000 \\
 PMT = \boxed{\rightarrow} -1454.01 \\
 FV = 0 \\
 P/Y = 12 \\
 C/Y = 2 \\
 PMT: \text{END} \text{ BEGIN}
 \end{array}$$

$$\begin{aligned}
 \text{Total paid} &= 1642.81 \times 20 \times 12 \\
 &= 394\,274.40
 \end{aligned}$$

$$\begin{aligned}
 &= 1454.01 \times 25 \times 12 \\
 &= 436\,203
 \end{aligned}$$

↑  
Better option  
if you can afford it.

Ex. 3 Given an interest rate of 5% for a mortgage of \$250 000, use your monthly payments from Ex. 2 (with amortization period of 25 years) and halve the amount. This will now be your bi-weekly and semi-monthly payments. Compare how long it will take to pay off the mortgage using bi-weekly vs. semi-monthly payments. Discuss why one frequency of payment is better than the other. Do you think the home owner will find a significant difference in the payments on a weekly basis?

$$\begin{aligned}
 &\frac{1454.01}{2} \\
 &= 727.01 \quad \text{bi-weekly (26)}
 \end{aligned}$$

semi-monthly (24)

$$\begin{array}{l}
 N = \boxed{\rightarrow} 558.43 \\
 I\% = 5 \\
 PV = 250000 \\
 PMT = -727.01 \\
 FV = 0 \\
 P/Y = 26 \\
 C/Y = 2 \\
 PMT: \text{END} \text{ BEGIN}
 \end{array}$$

$$\begin{array}{l}
 N = \boxed{\rightarrow} 598.8 \\
 I\% = 5 \\
 PV = 250000 \\
 PMT = -727.01 \\
 FV = 0 \\
 P/Y = 24 \\
 C/Y = 2 \\
 PMT: \text{END} \text{ BEGIN}
 \end{array}$$

$$\begin{aligned}
 N &= \# \text{yrs} \times \# \text{pmts} \\
 \# \text{yrs} &= \frac{N}{\# \text{pmts}} \\
 &= \frac{558.43}{26} \\
 &= 21.5
 \end{aligned}$$

$$\begin{aligned}
 \# \text{yrs} &= \frac{N}{\# \text{pmts}} \\
 &= \frac{598.8}{24} \\
 &= 24.95
 \end{aligned}$$

$$\begin{aligned}
 \text{Diff} &= 24.95 - 21.5 \\
 &= 3.5
 \end{aligned}$$

∴ Paying bi-weekly shaved  
3.5 years off the mortgage.

Ex. 4 Ms. Mes makes monthly payments on a \$ 72 000 mortgage over 25 years at 11.125% for 5 years. After 2 years, she decides to increase the monthly payment by \$100 and at the end of the 4<sup>th</sup> year she is able to make an extra principal payment of \$ 2000.

- ① Find monthly payment
- ② How much left after 2 yrs
- ③ How much left after 2 yrs at increased pmt.
- ④ Remaining after lump sum + 1 yr pmts.

a) What is the principal balance owing at the end of 5 yrs? 4 screens needed to complete!

①

|                                     |
|-------------------------------------|
| N=25×12                             |
| I%=11.125                           |
| PV=72000                            |
| PMT= <input type="text"/> → -699.21 |
| FV=0                                |
| P/Y=12                              |
| C/Y=2                               |
| PMT: <u>END</u> BEGIN               |

②

|                                      |
|--------------------------------------|
| N=2×12                               |
| I%=11.125                            |
| PV=72000                             |
| PMT=-699.21                          |
| FV= <input type="text"/> → -70754.91 |
| P/Y=12                               |
| C/Y=2                                |
| PMT: <u>END</u> BEGIN                |

After 2 yrs, loan is 70754.91

③

|                                      |
|--------------------------------------|
| N=2×12                               |
| I%=11.125                            |
| PV=70754.91                          |
| PMT=-799.21                          |
| FV= <input type="text"/> → -66541.22 |
| P/Y=12                               |
| C/Y=2                                |
| PMT: <u>END</u> BEGIN                |

④

|                                      |
|--------------------------------------|
| N=1×12                               |
| I%=11.125                            |
| PV=66541.22 - 2000                   |
| PMT=-799.21                          |
| FV= <input type="text"/> → -61837.82 |
| P/Y=12                               |
| C/Y=2                                |
| PMT: <u>END</u> BEGIN                |

∴ Her loan is worth \$61837.82 after 5 yrs

b) By how long has the amortization period of the mortgage been shortened?

|                                 |
|---------------------------------|
| N= <input type="text"/> → 133.9 |
| I%=11.125                       |
| PV=61837.82                     |
| PMT=-799.21                     |
| FV=0                            |
| P/Y=12                          |
| C/Y=2                           |
| PMT: <u>END</u> BEGIN           |

$$N = \#yrs \times \#pmts/yr$$

$$\frac{133.9}{12} = \#yrs$$

$$11.16 = \#yrs$$

$$Total = 11.16 + 5 = 16.16$$

Original loan was for 25 years!

$$Diff = 25 - 16.16 = 8.84$$

∴ She shortened her mortgage by 8.84 years!

**Homework**  
**Handout 7.5**  
**Using the TVM Solver for**  
**Mortgage Calculations**

Textbook always assumes monthly payments.

