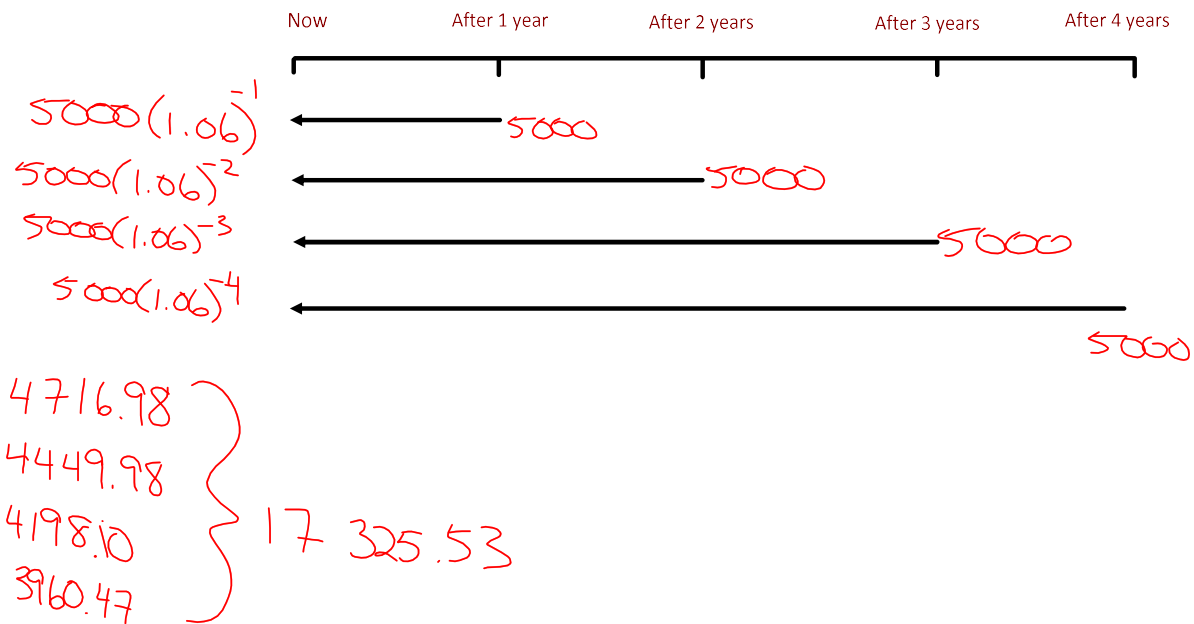


7.4 Present Value of an Annuity

Present Value of an Annuity: The amount of money (principal) that must be invested/borrowed NOW to provide a given series of equal payments at equal intervals of time.

Ex. 1 Next year, Jane is going back to university for a Ph.D. in psychology. She wants to know how much money to deposit now into an account that pays 6%/a, compounded annually, to provide a \$5000 payment each year for 4 years, with the first payment due a year from now.



Present Value of an Annuity Formula:

$$PV = R \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

- Use this to find the present value.

$$R = \frac{PVi}{\left[1 - (1 + i)^{-n} \right]}$$

- Use this to find the regular payment.

Don't worry we are going to give you all these formulas :)

where

- PV = Present Value
- R = Regular payment (made at the end of the compounding period)
- i = interest rate per compound pd.
- n = # of compound periods/# of payments (must be equal to use formula)

Ex. 1 James wants to invest now so that he will receive \$700 every month for 5 years. How much should he invest now at 4.3%/a compounded monthly to achieve this?

By Hand

Givens

$$R = 700$$

$$i = \frac{0.043}{12}$$

$$n = 12 \times 5 \\ = 60$$

$$PV = R \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

$$= 700 \left[\frac{1 - \left(1 + \frac{0.043}{12}\right)^{-60}}{\frac{0.043}{12}} \right]$$

$$= 37731.35$$

∴ The present value is
\$37731.35

b) How much interest did he earn?

$$\text{Final amount} = \$700 \times 60 \\ = 42000$$

He only paid \$37731.35

$$\therefore \text{Interest is } 42000 - 37731.35 \\ = 4268.65$$

By TVM

$$N = 60$$

$$I\% = 4.3$$

$$PV = \square \rightarrow -37731.35$$

$$PMT = 700$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

$$PMT: \text{END} \text{ BEGIN}$$

7.4 Present Value of an Annuity2.notebook

June 02, 2023

Ex. 2 Charlie has won the lottery prize of a lump sum payment of \$78 000. He has placed the money into an account at 6.3%/a compounded semi-annually and plans to withdraw an equal payment every 6 months for 10 years. How big will the payment be? ($\$5315.80$)

By Hand

Given

$$PV = 78000$$

$$i = \frac{0.063}{2}$$

$$n = 2 \times 10 \\ = 20$$

$$R = \frac{PV \cdot i}{\left[1 - (1+i)^{-n}\right]}$$
$$= \frac{78000 \left(\frac{0.063}{2}\right)}{\left[1 - \left(1 + \frac{0.063}{2}\right)^{-20}\right]}$$
$$= 5315.80$$

\therefore The payment will be
 $\$5315.80$

b) How much interest has he earned?

$$\text{Final amount: } 5315.80 \times 20 \\ = 106316$$

Only paid 78000

$$\therefore \text{Interest} = 106316 - 78000 \\ = 28316$$

By TVM

N= 20
I%= 6.3
PV= -78000
PMT= $\square \rightarrow 5315.80$
FV= 0
P/Y= 2
C/Y= 2
PMT: <u>END</u> BEGIN



Homework
Pg. 461
#C2,3bc,
4,6-9,11,12,14
(Graphing Calculator
4,6,9)