BINOMIAL PATTERN

Try expanding the expression $(1 + a)^n$ for various values of n. Do your rough work elsewhere, and fill in only the final answers below.

$(1 + a)^0 =$ ________

$(1 + a)^1 =$ _____________

$(1 + a)^2 =$ __________________

$(1 + a)^3 =$ _______________________

$(1 + a)^4 =$ ____________________________

$(1 + a)^5 =$ _________________________________

$(1 + a)^6 =$ ______________________________________

Now take a coloured marker and rewrite the coefficients (including any "ones" which we normally don't write).

Compare the value of n with the row in Pascal's Triangle which contains the coefficients.

Rewrite each of the above lines, letting a=1.
Can you make any conclusion?

Compare your answer with the ants in the maze...

The Binomial Theorem

7. Using the copy of Pascal's Triangle you completed in Exercise 3.1, Question 5, expand each of the following.
   (a) $(a+b)^9$
   (b) $(a+b)^{10}$
   (c) $(a+b)^{11}$

1. Without simplifying, state the terms in the expansion of each of the following.
   ~~(a) $(a+b)^5$~~
   (b) $(x+y)^3$
   (c) $\left(\frac{1}{4}+\frac{3}{4}\right)^4$

2. If the numerical coefficients are disregarded, terms of the following form will appear in the expansion of $(a+b)^9$. State the value of the exponent k in each case.
   (a) a^3b^k
   (b) a^kb^8
   (c) a^9b^k
   (d) $a^{k+1}b^k$

5. Expand and simplify each of the following.
   (a) $(2a+b)^3$ (b) $(a-2b)^4$
   (c) $(1-x)^5$ (d) $(1+x^2)^6$
   (e) $\left(1+\frac{1}{x}\right)^4$ (f) $\left(x-\frac{1}{x}\right)^5$

6. Expand and simplify each of the following.
   (a) $\left(x-\frac{2}{x^2}\right)^5$ (b) $\left(2x^3+\sqrt{y}\right)^4$
   (c) $\left(a^2+\frac{3b}{a}\right)^4$ d) $\left(\sqrt{x}-\frac{2}{\sqrt{x}}\right)^6$

7. Find the first four terms in the expansion of each of the following.
   (a) $(a+b)^{10}$ (b) $(1-x^2)^{12}$
   (c) $\left(x^2+\frac{2}{x^2}\right)^9$ (d) $\left(2x-\frac{3}{x^2}\right)^8$
   (e) $\left(x^3-\frac{2}{x^2}\right)^6$ (f) $\left(x+\sqrt{x^3}\right)^{11}$

C 14. In the expansion of $(1+x)^n$, the first three terms are $1-18+144$. Find the values of x and n.

15. In the expansion of $(1+ax)^n$, the first three terms are $1+\frac{5}{3}x+\frac{10}{9}x^2$. Find the value of a and n.

16. The polynomial $(p+q)^9$ is expanded in decreasing powers of p. The second and third terms have equal values, where p and q are positive numbers whose sum is one. What is the value of p?

Answers:

7. (a) $a^9+9a^8b+36a^7b^2+84a^6b^3+126a^5b^4+126a^4b^5+84a^3b^6+36a^2b^7+9ab^8+b^9$
(b) $a^{10}+10a^9b+45a^8b^2+120a^7b^3+210a^6b^4+252a^5b^5+210a^4b^6+120a^3b^7+45a^2b^8+10ab^9+b^{10}$ (c) $a^{11}+11a^{10}b+55a^9b^2+165a^8b^3+330a^7b^4+462a^6b^5+462a^5b^6+330a^4b^7+165a^3b^8+55a^2b^9+11ab^{10}+b^{11}$

1. (a) $\binom{5}{0}a^5+\binom{5}{1}a^4b+\binom{5}{2}a^3b^2+\binom{5}{3}a^2b^3+\binom{5}{4}ab^4+\binom{5}{5}b^5$ (b) $\binom{3}{0}x^3+\binom{3}{1}x^2y+\binom{3}{2}xy^2+\binom{3}{3}y^3$ (c) $\binom{4}{0}\left(\frac{1}{4}\right)^4+\binom{4}{1}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)+\binom{4}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^2+\binom{4}{3}\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^3+\binom{4}{4}\left(\frac{3}{4}\right)^4$

2. (a) 6 (b) 1 (c) 0 (d) 4

5. (a) $8a^3+12a^2b+6ab^2+b^3$
(b) $a^4-8a^3b+24a^2b^2-32ab^3+16b^4$
(c) $1-5x+10x^2-10x^3+5x^4-x^5$
(d) $1+6x^2+15x^4+20x^6+15x^8+6x^{10}+x^{12}$
(e) $1+\frac{4}{x}+\frac{6}{x^2}+\frac{4}{x^3}+\frac{1}{x^4}$
(f) $x^5-5x^3+10x-\frac{10}{x}+\frac{5}{x^3}-\frac{1}{x^5}$

6. (a) $x^5-10x^2+\frac{40}{x}-\frac{80}{x^4}+\frac{80}{x^7}-\frac{32}{x^{10}}$
(b) $16x^{12}+32x^9\sqrt{y}+24x^6y+8x^3\sqrt{y^3}+y^2$
(c) $a^8+12a^5b+54a^2b^2+\frac{108b^3}{a}+\frac{81b^4}{a^4}$
(d) $x^3-12x^2+60x-160+\frac{240}{x}-\frac{192}{x^2}+\frac{64}{x^3}$

7. (a) $a^{10}+10a^9b+45a^8b^2+120a^7b^3+\ldots$
(b) $1-12x^2+66x^4-220x^6+\ldots$
(c) $x^{18}+18x^{14}+144x^{10}+672x^6+\ldots$
(d) $256x^8-3072x^5+16\,128x^2-\frac{48\,384}{x}+\ldots$
(e) $x^{18}-12x^{13}+60x^8-160x^3+\ldots$
(f) $x^{11}+11\sqrt{x^{23}}+55x^{12}+165\sqrt{x^{25}}+\ldots$

14. $n=9$, $x=-2$

15. $n=5$, $a=\frac{1}{3}$ 16. $\frac{4}{5}$