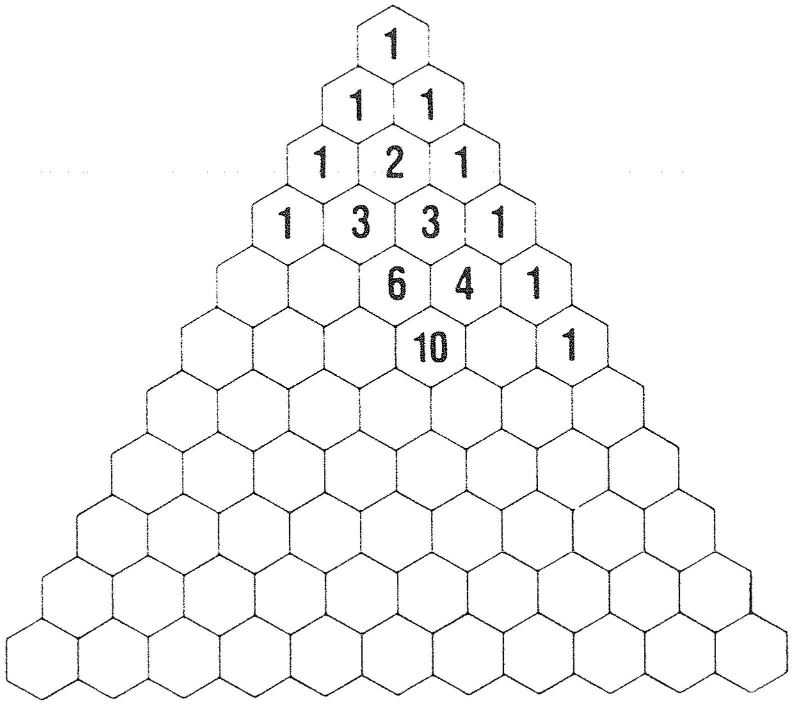
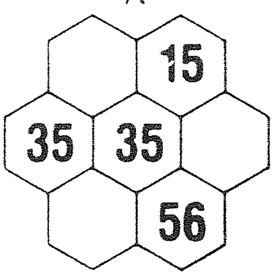
WORKSHEET 1

Use the pattern to fill in the missing numbers in Pascal's triangle.



Shown below are portions of Pascal's triangle. Fill in the missing numbers.

A



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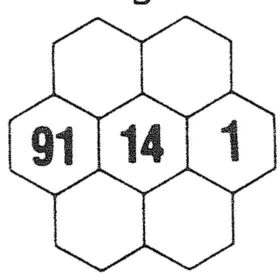

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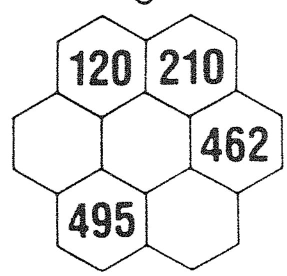

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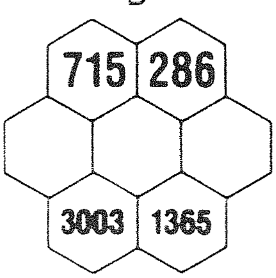

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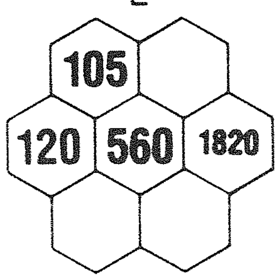

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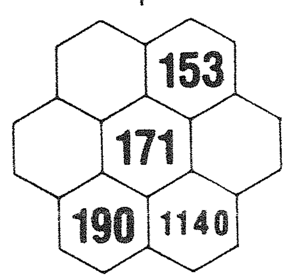


Patterns in Pascal's Triangle

5. Make a copy of Pascal's Triangle to include the rows where $n=0$ through $n=12$. Keep this handy as it will be useful in the questions that follow and in future sections.

There are many other patterns that arise from Pascal's Triangle. The following questions are designed to help you discover some of these patterns.

6. (a) Add the numbers in each row of Pascal's Triangle and complete a chart similar to the one given.

n	Sum of the Numbers in the Row	
0	1	= 1
1	1 + 1	= 2
2	1 + 2 + 1	= 4
⋮	⋮	
6		

(b) What would you predict to be the sum of the numbers in the row where $n=7$? Check your prediction.

11. (a) Add the squares of the numbers in each row of Pascal's Triangle and complete a chart similar to the one given.

n	Sum of the Squares of the Numbers in the Row	
0	1^2	= 1
1	$1^2 + 1^2$	= 2
2	$1^2 + 2^2 + 1^2$	= 6
⋮	⋮	
5		

(b) Locate these numbers in Pascal's Triangle.

(c) What would you predict to be the sum of the squares of the numbers in the row where $n=6$? Check your prediction.

13. In each row of Pascal's Triangle, alternate the signs of each entry; the first positive, the second negative, the third positive, the fourth negative, and so on.

(a) Find the sum of the new elements in each row and complete a chart similar to the one given.

n	Sum of the Numbers in the Row (Alternate Signs Assumed)	
0	1	= 1
1	1 − 1	= 0
2	1 − 2 + 1	= 0
⋮	⋮	
6		

(b) What would you predict to be the sum of the numbers in the row where $n=7$? Check your prediction.

5. $n=12$: 1 12 66 220 495 792 924 792 495 220 66 12 1

6. (b) 128 (c) (i) 256 (ii) 512 (iii) 1024 (d) 2^n

13. (b) 0 (c) (i) 0 (ii) 0 (d) $\begin{cases} 1 & \text{if } n=0 \\ 0 & \text{if } n \neq 0 \end{cases}$

1. In the arrangement of the letters given, starting from the top we proceed to the row below by moving diagonally to the immediate right or left. How many different paths will spell each of the following names?

(a) PASCAL

```
                P
             A     A
          S     S     S
       C     C     C     C
    A     A     A     A     A
 L     L     L     L     L     L
```

(b) BLAISE

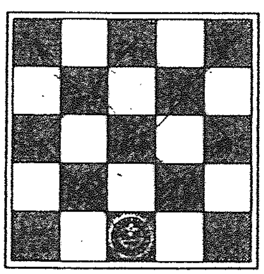
```
          B
       L     L
    A     A     A
 I     I     I     I
    S     S     S
       E     E
```

(c) EULER

```
       E
    U     U
       L
    E     E
       R
```

(d) Your last name if the letters are placed in an arrangement similar to part (a).

2. Consider the five-square by five-square gameboard given.

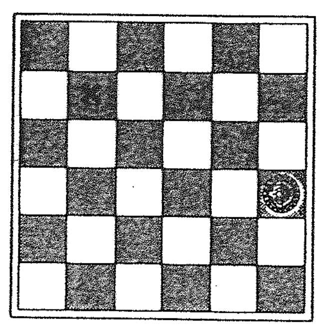


A checker is positioned in the middle of the bottom row. The checker is allowed to move one square at a time, diagonally left or right, to the row above. How many different paths will lead to each of the positions in the top row?

3 Repeat Question 2 for a checker positioned in the bottom left corner.

4. In Question 2, assume the checker is allowed to move one square diagonally left or right or straight forward. How many different paths will lead to each of the positions in the top row?

5. Consider a six-square by six-square gameboard. Your piece is in the bottom row and the opponent's piece is indicated by X.



The opponent's piece does not move. Your piece is allowed to move one square at a time diagonally left or right, unless the opponent's piece is in the way. In that case, your piece is allowed to jump over the opponent's piece to the square two rows away. How many different paths will lead to each position in the top row?

6. A store is located six blocks east and four blocks south of your house. If yc re allowed to walk only east or south, how many different paths lead from your house to the store?

1. (a) 32 (b) 20 (c) 4
2. 3,0,6,0,3
3. 2,0,3,0,1
4. 9,16,19,16,9
5. 0,9,0,7,0,3
6. 210

14. A different way of writing the numbers in Pascal's Triangle is to position the first entries in the rows in a column, the second entries in the rows in another column, and so on.

r / *n*	0	1	2	3	4	5	6	...
0	1							
1	1	1						
2	1	2	1					
3	1	3	3	1				
4								
5								
6								
⋮								

Extend a chart similar to the one given through nine rows. Make reference to this chart when answering Questions 15 and 17.

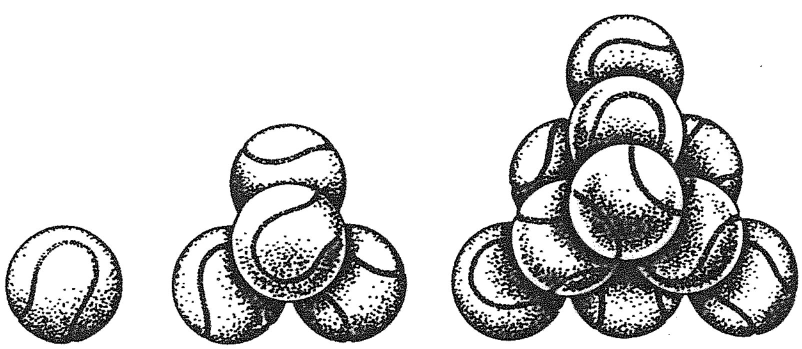
15. Quarters can be arranged in the shape of an equilateral triangle as shown.

The first contains one quarter, the second contains three quarters, the third, six quarters, and so on.

(a) How many quarters do the fourth, fifth, and sixth contain?

(b) Such numbers are called **triangular numbers**. [Note: The *n*th triangular number is the sum of the first *n* natural numbers.] Locate the triangular numbers in the version of Pascal's Triangle you made in Question 14.

17. Tennis balls can be arranged in the shape of a tetrahedron as shown.



The first contains one tennis ball, the second contains four tennis balls, the third, ten, and so on.

(a) How many tennis balls do the fourth, fifth, and sixth contain?

(b) Such numbers are called **tetrahedral numbers**. Locate the tetrahedral numbers in the version of Pascal's Triangle you made in Question 14.

15 ⓐ 10, 15, 21
17 ⓐ 20, 35, 56