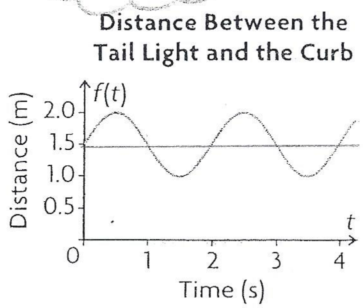
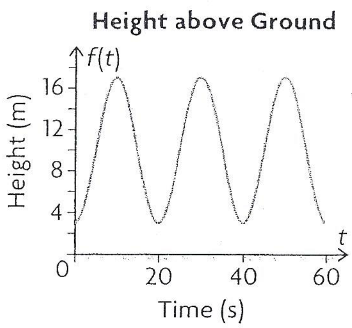
(5.6)





CHECK Your Understanding

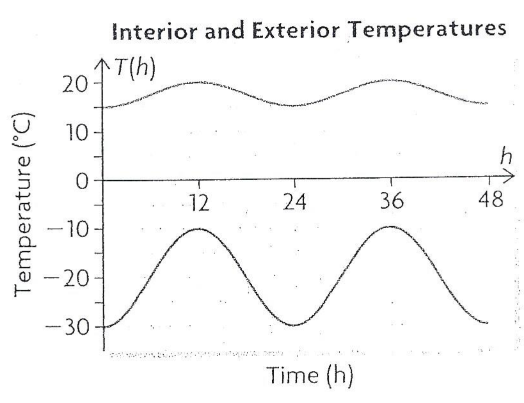
1. The load on a trailer has shifted, causing the rear end of the trailer to swing left and right. The distance from one of the tail lights on the trailer to the curb varies sinusoidally with time. The graph models this behaviour.
   a) What is the equation of the axis of the function, and what does it represent in this situation?
   b) What is the amplitude of the function, and what does it represent in this situation?
   c) What is the period of the function, and what does it represent in this situation?
   d) Determine the equation and the range of the sinusoidal function.
   e) What are the domain and range of the function in terms of the situation?
   f) How far is the tail light from the curve at $t = 3.2$ s?

2. Don Quixote, a fictional character in a Spanish novel, attacked windmills because he thought they were giants. At one point, he got snagged by one of the blades and was hoisted into the air. The graph shows his height above ground in terms of time.
   a) What is the equation of the axis of the function, and what does it represent in this situation?
   b) What is the amplitude of the function, and what does it represent in this situation?
   c) What is the period of the function, and what does it represent in this situation?
   d) If Don Quixote remains snagged for seven complete cycles, determine the domain and range of the function.
   e) Determine the equation of the sinusoidal function.
   f) If the wind speed decreased, how would that affect the graph of the sinusoidal function?

3. Chantelle is swinging back and forth on a trapeze. Her distance from a vertical support beam in terms of time can be modelled by a sinusoidal function. At 1 s, she is the maximum distance from the beam, 12 m. At 3 s, she is the minimum distance from the beam, 4 m. Determine an equation of a sinusoidal function that describes Chantelle's distance from the vertical beam in relation to time.

PRACTISING

4. **K** The interior and exterior temperatures of an igloo were recorded over a 48 h period. The data were collected and plotted, and two curves were drawn through the appropriate points.



5.6 (continued)

a) How are these curves similar? Explain how each of them might be related to this situation.

b) Describe the domain and range of each curve.

c) Assuming that the curves can be represented by sinusoidal functions, determine the equation of each function.

5. Skyscrapers sway in high-wind conditions. In one case, at $t = 2$ s, the top floor of a building swayed 30 cm to the left (-30 cm), and at $t = 12$, the top floor swayed 30 cm to the right $(+30 \text{ cm})$ of its starting position.

a) What is the equation of a sinusoidal function that describes the motion of the building in terms of time?

b) Dampers, in the forms of large tanks of water, are often added to the top floors of skyscrapers to reduce the severity of the sways. If a damper is added to this building, it will reduce the sway (not the period) by 70%. What is the equation of the new function that describes the motion of the building in terms of time?

6. Milton is floating in an inner tube in a wave pool. He rises 1.5 m from the bottom of the pool when he is at the trough of a wave. A stopwatch starts timing at this point. In 1.25 s, he is on the crest of the wave, 2.1 m from the bottom of the pool.

a) Determine the equation of the function that expresses Milton's distance from the bottom of the pool in terms of time.

Answers

1\. a) $d = 1.5$ m, distance between tail lights and the curb if the trailer isn't swinging back and forth

b) amplitude: 0.5 m, distance the trailer swings to the left and right

c) period: 2 s, the time it takes for the trailer to swing back and forth

d) ~~$d = 0.5\ (180t)^\circ + 1.5$~~ $\{d \in R \mid 1 \le d \le 2\}$ $d = 0.5\sin(180t)^\circ + 1.5$

e) range is the distance the trailer swings back and forth; domain is time

f) 1.2 m

2\. a) $h = 10$ m, axle height

b) amplitude: 7 m, length of blade

c) period: 20 s, time in seconds to complete revolution

d) domain: $\{t \in R \mid 0 \le t \le 140\}$;
range: $\{h \in R \mid 3 \le h \le 17\}$

e) $h = -7\cos(18x)^\circ + 10$

f) period would be larger

3\. $d = 4\cos(90(t-1))^\circ + 8$

4\. a) same period (24), same horizontal translation (12), different amplitude (2.5 and 10), different equations of the axis ($T = 17.5$ and $T = -20$). The top one is probably the interior temperature (higher, with less fluctuation).

b) domain (for both): $\{t \in R \mid 0 \le t \le 48\}$;
range (top): $\{T \in R \mid 15 \le T \le 20\}$;
range: (bottom): $\{T \in R \mid -30 \le T \le -10\}$

c) blue: $T = 2.5\cos(15(h-12))^\circ + 17.5$;
red: $T = 10\cos(15(h-12))^\circ - 20$

5\. a) ~~$d = -30\cos(18t)^\circ$~~ $d = -30\cos[18(t-2)]^\circ$

b) ~~$d = 9\cos(36(t-12))^\circ$~~ $d = -9\cos[18(t-2)]^\circ$

6\. a) $d = -0.3\cos(144t)^\circ + 1.8$

b) amplitude: 0.3, height of crest relative to normal water level

c) 2 m

d) 16

e) $d = -0.3\cos(120t)^\circ + 1.8$