

7.2 Compound Interest

Present Value (P or PV):

- Principal that needs to be invested/borrowed now to achieve a future goal.
- PV can be calculated when the interest rate, compounding period and length of term are known.

Present Value Formula: Use compound interest formula, rearranged for "P".

$$A = P(1+i)^n$$

$$\frac{A}{(1+i)^n} = P$$

$$A(1+i)^{-n} = P$$

where
 A = Amount at end of investment (\$)
 P = Present value/principal (\$)
 i = Interest rate per compound pd.
 n = # of compound periods

Ex. 1 Julia wants to have \$5000 in 2 years to use as a down-payment for a car. How much does she need to invest now at 6.3%/a compounded monthly?

Given
 P = ?
 A = 5000
 $i = \frac{0.063}{12}$
 n = 2 x 12 = 24

$$P = \frac{A}{(1+i)^n}$$

$$= \frac{5000}{(1 + \frac{0.063}{12})^{24}}$$

$$= 4409.53$$

∴ She will need to invest \$4409.53

Ex. 2 David plans to put money into an RESP so that he has \$9000 in 5 years. Which option is the best deal for David?

<p>a) 7.2% compounded semi-annually</p> <p>Given A = 9000 $i = \frac{0.072}{2}$ n = 2 x 5 = 10</p>	$P = \frac{A}{(1+i)^n}$ $= \frac{9000}{(1 + \frac{0.072}{2})^{10}}$ $= 6318.95$	<p>b) 6.5% compounded bi-weekly</p> <p>Given A = 9000 $i = \frac{0.065}{26}$ n = 26 x 5 = 130</p>	$P = \frac{9000}{(1 + \frac{0.065}{26})^{130}}$ $= 6505.38$
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↑
 "Put in" least amount
 ∴ Option A is the best choice

The TVM Solver

A program on the graphing calculator used for financial calculations.

To find the TVM Solver program:

APPS, 1: Finance..., **ENTER**, 1: TVM Solver... **ENTER**

N = # of years
 I% = interest rate/a as a percent
 PV = present value (P)
 PMT = the payment amount (put as "0" if there are no payments)
 FV = future value (A)
 P/Y = number of payments per year (put as "1" if there are no payments)
 C/Y = number of compound periods per year
 PMT: = choose **END**

To determine a value:

- Move the cursor to the appropriate line and press **ALPHA** **ENTER**

Ex. 3 Harriet invested ^{PV} \$4 000 in an investment fund that pays 6.5%/a compounded monthly. How much money is in the account after 8 years?

C/Y = 12


n = 8

N = 8

I% = 6.5 interest rate/a as a percent

PV = - 4000 cash out neg-money going out of your pocket into the bank

PMT = 0 no regular payments-this is a one time investment

FV =  move cursor to here then press **ALPHA** **ENTER** = \$ 6718.68

P/Y = 1 Put as a 1 as no payments


C/Y = 12 number of compound periods per year

PMT: **END** BEGIN

Ex. 4 How much do you need to invest now at 8.2%/a compounded quarterly to have \$20 000 in 5 years?

N = 5

I% = 8.2

PV =  - 13328.15

PMT = 0

FV = 20 000

P/Y = 1

C/Y = 4

PMT: **END** BEGIN

∴ You need to invest \$13328.15

Ex. 4 Graham deposits \$4500 into an account paying 3.5%/a compounded monthly. How long will it take him to have \$8000 to buy a motorcycle?

$$\begin{aligned}
 N &= \square \rightarrow 16.5 \\
 I\% &= 3.5 \\
 PV &= -4500 \\
 PMT &= 0 \\
 FV &= 8000 \\
 P/Y &= 1 \\
 C/Y &= 12 \\
 PMT &: \text{END} \text{ BEGIN}
 \end{aligned}$$

∴ It will take 16.5 yrs

Ex. 5 Jason invested \$1200 in a fund that compounded interest semi-annually. At what rate did he invest at if he earned \$500 interest in 6 years?

$$\begin{aligned}
 N &= 6 \\
 I\% &= \square \rightarrow 5.9 \\
 PV &= -1200 \\
 PMT &= 0 \\
 FV &= 1200 + 500 = 1700 \\
 P/Y &= 1 \\
 C/Y &= 2 \\
 PMT &: \text{END} \text{ BEGIN}
 \end{aligned}$$

∴ He will need to invest at 5.9%

Ex. 6 Which investment will reach \$10 000 faster?

A: \$7500 invested at 4.5%/a compounded monthly

OR

B: \$8200 invested at 4.2% compounded semi-annually

$$\begin{aligned}
 N &= \square \rightarrow 6.4 \\
 I\% &= 4.5 \\
 PV &= -7500 \\
 PMT &= 0 \\
 FV &= 10000 \\
 P/Y &= 1 \\
 C/Y &= 12 \\
 PMT &: \text{END} \text{ BEGIN}
 \end{aligned}$$

$$\begin{aligned}
 N &= \square \rightarrow 4.8 \\
 I\% &= 4.2 \\
 PV &= -8200 \\
 PMT &= 0 \\
 FV &= 10000 \\
 P/Y &= 1 \\
 C/Y &= 2 \\
 PMT &: \text{END} \text{ BEGIN}
 \end{aligned}$$

↑ 4.8 yrs!
This choice is faster

Ex. 7 Go back to examples 1 and 2 and use the TVM solver to check your answers.

Ex. 8 Brian is investing \$6800 at an interest rate of 7% per annum, compounded quarterly, for 2 years. Then, he will invest the amount plus additional money at 6.5%/a, compounded semi-annually, for 3 years. At the end of the second investment, he wants to have \$15 000. How much extra must he invest for the second investment?

N=2
I%=7
PV=-6800
PMT=0
FV= <input type="checkbox"/> → 7812.40
P/Y=1
C/Y=4
PMT: <u>END</u> BEGIN

N=3
I%=6.5
PV= <input type="checkbox"/> -12380.86
PMT=0
FV=15000
P/Y=1
C/Y=2
PMT: <u>END</u> BEGIN

How much more than 7812.40 is 12380.86?

$$\begin{aligned} \text{difference} &= 12380.86 - 7812.40 \\ &= 4568.46 \end{aligned}$$

∴ He will need to invest an additional \$4568.46

Homework
Page 441 #C2,1b,2,4-6,8,9,11,14,17



<https://www.youtube.com/watch?v=TN7tM7iOx4E> Virtual TI for Mac users.