7.1 Simple and Compound Interest

Simple Interest: Interest is earned only on the original investment.

Simple Interest Formulas:

$$I = Prt \qquad A = P + I$$

where
A = Amount at the end of investment ($)
P = Principal or original amount ($)
r = Rate of interest per year (decimal)
t = Time invested (years)
I = Total interest earned ($)

Ex. 1 Veeta invests $900 at 5%/a for 7 years. Per Annum "Year"

a) How much interest does she earn?

b) What is the total amount in the account?

Given
$P = 900$
$r = 0.05$
$t = 7$

a) $I = Prt$
$= 900(0.05)(7)$
$= 315$
∴ She earns $315

b) $A = P + I$
$= 900 + 315$
$= 1215$
∴ She has $1215

Ex. 2 Margot invests $100 at 7%/ a for 5 years.

a) Complete the table to examine what happens to her investment.

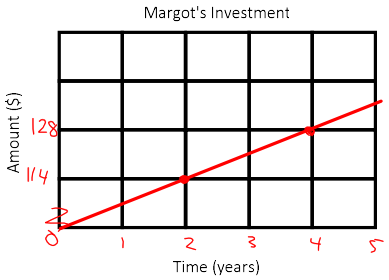
Year	Interest ($)	Amount ($)
0	✕	100
1	7	107
2	7	114
3	7	121
4	7	128
5	7	135

+7, +7

What type of sequence does this represent?

Arithmetic! LINEAR

b) Sketch the growth of her money over the 5 years.



What type of growth does this represent?

Interest is constant: $ 7 / yr
Slope is 7

Simple Interest:
- Increases by the same amount of money for each time interval.
- Creates an Arithmetic sequence.
- Represents Linear growth.

Compound Interest: http://time.com/money/4343323/compound-interest-returns

- **Interest is added to the principal for the next compound period.**
- **Has the effect of paying/earning interest on interest.**

Ex. 1 Consider Margot's investment of $100 at 7% if the interest is compounded yearly.

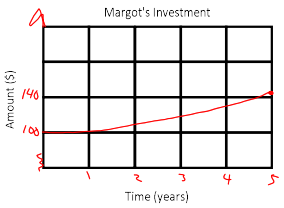
a) Complete the table to examine what happens to her investment.

Year	Interest ($)	Amount ($)
0		100
1	7% of 100 = 7	107
2	7% of 107 = 7.49	114.49
3	0.07 × 114.49 = 8.01	122.50
4	0.07 × 122.5 = 8.58	131.08
5	0.07 × 131.08 = 9.18	140.26

×1.07

What type of sequence does this represent? Geometric

b) Sketch the growth of her money over the 5 years.



What type of growth does this represent? Exponential.

Amount has a constant ratio
r = 1.07

Compound Interest:
- Increases by a constant multipier for each compound period.
- Creates a geometric sequence.
- Represents exponential growth.

Compound Interest Formulas:

$A = P(1 + i)^n$ $I = A - P$

This is the formula for exponential growth. The growth factor is $(1 + i)$.

where

P = Principal or amount invested/borrowed ($)
A = Amount at the end of the investment ($)
i = Interest rate per compound period (decimal)

$i = \frac{\text{rate}}{\text{\# of compounding periods per year}}$

n = Number of compound periods

n = (# of years) x (# of periods per year)

Compounding Periods --> How often interest is compounded.

Typical compound periods	# of compounds / year
annually	1
semi-annually	2
quarterly	4
monthly	12
bi-monthly (*twice a month*)	24
weekly	52
bi-weekly	26
daily	365

8%/a compounded quarterly
r = 0.08
$i = \frac{0.08}{4}$
= 0.02

10%/a compounded bimonthly
r = 0.1
$i = \frac{0.1}{24}$
≈ 0.0042

Ex. 2 Myla invests $1500 in an account paying 4.75%/a compounded quarterly. How much money will she have at the end of 5 years?

Given
$P = 1500$
$r = 4.75$
$i = \frac{0.0475}{4}$
$n = 4 \text{ cmpds} \times 5 \text{ years}$
$= 20$

$A = P(1+i)^n$
$= 1500\left(1+\frac{0.0475}{4}\right)^{20}$
$= 1899.45$
∴ She will have $1899.45

Ex. 3 Sarah needs to borrow $4500 to buy her first car.
(She will not be making payments but will pay it off in one lump sum in 5 years.)
She has 2 options:
a) 3.4 %/a for 5 years compounded monthly OR
b) 3.9%/a for 5 years compounded semi-annually.

Which option is better?

Given
$P = 4500$
$r = 0.034$
$i = \frac{0.034}{12}$
$n = 12 \times 5$
$= 60$

$A = 4500\left(1+\frac{0.034}{12}\right)^{60}$
$\doteq 5332.59$

Less to PAY!
∴ Option A is better

Given
$P = 4500$
$r = 0.039$
$i = \frac{0.039}{2}$
$n = 2 \times 5$
$= 10$

$A = 4500\left(1+\frac{0.039}{2}\right)^{10}$
$\doteq 5458.64$

Ex. 4 Don has $24 000 invested in a University fund that he hopes will grow to $30 000 in 3 years. What interest rate, compounded quarterly will he need to invest at in order to achieve his goal?

Given
$P = 24000$
$A = 30000$
$n = 4 \times 3$
$= 12$

$A = P(1+i)^n$
$30000 = 24000(1+i)^{12}$
$\frac{30000}{24000} = (1+i)^{12}$
$1.25 = (1+i)^{12}$
$\sqrt[12]{1.25} = 1+i$
$i = \sqrt[12]{1.25} - 1$
$\doteq 0.01877$

i of 0.01877 is QUARTERLY!
$r = 0.01877 \times 4$
$\doteq 7.5\%$

∴ The annual interest rate is 7.5%

HOMEWORK
Pg. 423 # C3,3,5,8

Pg. 433 # 1,3d,5c,6,9,11,14
OR
EXTRA WORKSHEET ON WEBSITE