6.5 - Geometric Series

4, 8, 16, 32, ... geometric sequence

4 + 8 + 16 + 32 + ... geometric series: The **sum** of the terms of a geometric sequence.

Derivation of the Geometric Series Formula

$$S_n = a + ar + ar^2 + ... + ar^{n-1}$$
$$rS_n = \quad ar + ar^2 + ... + ar^{n-1} + ar^n$$

$$S_n - rS_n = a + (-ar^n)$$
$$S_n(1-r) = a - ar^n$$
$$S_n(1-r) = a(1-r^n)$$
$$S_n = \frac{a(1-r^n)}{1-r}$$

Geometric Series Formulas

Any term, t_n, can be found using: $t_n = a\,r^{n-1}$

Any sum, S_n, can be found using:

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{OR} \quad \frac{a(1 - r^n)}{1 - r}, \text{ where } r \neq 1$$

and:
a = first term
r = common ratio
n = # of terms

Ex. 1 Determine the indicated sum of each series.

a) $4 - 8 + 16 - \ldots$, S_9

$r = \frac{-8}{4}$

$= -2$

$a = 4$

$n = 9$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

$$S_9 = \frac{4(1-(-2)^9)}{(1-(-2))}$$

$$= \frac{4(1-(-512))}{3}$$

$$= \frac{4(513)}{3}$$

$$= 684$$

b) $64 + 32 + 16 + \ldots$, S_{12}

$r = \frac{1}{2}$

$a = 64$

$n = 12$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{12} = \frac{64\left(1-\left(\frac{1}{2}\right)^{12}\right)}{1-\frac{1}{2}}$$

$$= \frac{64\left(1-\frac{1}{4096}\right)}{\frac{1}{2}}$$

$$= 64\left(\frac{4095}{4096}\right) \div \frac{1}{2}$$

$$= \frac{4095}{64} \times \frac{2}{1}$$

$$= \frac{4095}{32}$$

$\frac{4095}{32}$

Ex. 2 Determine the sum of the series.

$2 + 6 + 18 + \ldots + 4374$

$r = \frac{t_2}{t_1}$

$= 3$

$a = 2$

$n = ?$

Find n

$t_n = ar^{n-1}$

$4374 = 2(3)^{n-1}$

$2187 = 3^{n-1}$

$3^7 = 3^{n-1}$

$\therefore 7 = n - 1$

$8 = n$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_8 = \frac{2(1-3^8)}{1-3}$

$= \frac{2(-6560)}{-2}$

$= 6560$

$\therefore S_8 = 6560$

Ex. 3 What if r = 1? Suppose a = 5 and r =1, find S_{10} for the series.

$5 + 5 + 5 + 5 + \ldots$

$S_n = \frac{a(1-r^n)}{1-r}$

← Can't divide by zero!

$S_{10} = 5 \cdot 10$

$= 50$

Homework
p. 407 #C3, 2abdf, 3bf, 5bd, 6-12, 16