6.4 - Arithmetic Series

Arithmetic Sequence: 2, 5, 8, 11, 14, ...

Arithmetic Series: 2 + 5 + 8 + 11 + 14 + ...

An arithmetic series is the SUM of the terms in an arithmetic sequence.

S_n represents the sum of the first n terms

eg. For 2 + 5 + 8 + 11 + ...

$S_2 = t_1 + t_2$
$= 2 + 5$
$= 7$

$S_3 = t_1 + t_2 + t_3$
$= 2 + 5 + 8$
$= 15$

OR $S_3 = S_2 + t_3$

Johann Carl Friedrich Gauss
German mathematician

Development of the Arithmetic Series Formula:

Gauss added the numbers from 1 to 100 by:

1 + 2 + 3 + ... + 99 + 100
100 + 99 + 98 + ... + 2 + 1

101 + 101 + 101 + ... + 101 + 101

Each sum would = 101

There would be 100 sums all together... but this is two of the series added together so the sum is twice what it should be.

$$S_{100} = \frac{100(101)}{2}$$
$$= 5050$$

Try it out! Nice party trick :)

In general, an arithmetic sequence is: a, a + d, a + 2d, + ... + , t_n - d, t_n

So, in general, the sum of an arithmetic sequence is:

$$S_n = a + (a + d) + (a + 2d) + \dots + (t_n - d) + t_n$$
$$S_n = t_n + t_n - d + t_n - 2d + \dots + (a+d) + a$$

$$2S_n = (a+t_n) + (a+t_n) + (a+t_n) + \dots + (a+t_n) + (a+t_n)$$

$$2S_n = n(a+t_n)$$
$$S_n = \frac{n}{2}(a+t_n) \qquad t_n = a+(n-1)d$$

$$\frac{n}{2}(a+a+(n-1)d)$$
$$= \frac{n}{2}(2a+(n-1)d)$$

OR

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Arithmetic Series Formulas

Any term in an arithmetic sequence/series , t_n can be found using:

$$t_n = a + (n - 1)d$$

Any sum in an arithmetic series, S_n can be found using:

$$S_n = \frac{n(a+t_n)}{2} \quad \text{or} \quad S_n = \frac{n}{2}[2a+(n-1)d]$$

Use when you know t_n

Use when you know d

Ex. 1 Find the indicated sum for each series.

a) $4 + 6 + 8 + 10 + \ldots$ S_{42}

Givens

$a = 4$

$d = 2$

$n = 42$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{42}{2}\left[2(4) + (42-1)(2)\right]$

$= 21(8 + 41 \cdot 2)$

$= 1890$

b) $5 - 3 - 11 - 19 - \ldots$, S_{17}

$a = 5$

$d = (-3) - (5)$

$= -8$

$n = 17$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{17}{2}\left[2(5) + (17-1)(-8)\right]$

$= \frac{17}{2}(-118)$

$= -1003$

Ex. 2 Find the sum of the series.

a) $2 + 5 + 8 + 11 + \ldots + 254$

Givens

$a = 2$

$d = 3$

To find n

$t_n = a + (n-1)d$

Sub a, d, and $t_n = 254$

$254 = 2 + (n-1)(3)$

$252 = 3n - 3$

$255 = 3n$

$85 = n$

$S_n = \frac{n}{2}(a + t_n)$

$= \frac{85}{2}(2 + 254)$

$= \frac{85}{2}(256)$

$= 10880$

$\therefore S_{85} = 10880$

b) $5 + 3 + 1 - 1 - \ldots - 401$

(-40392)

$a = 5$

$d = -2$

To find n

$-401 = 5 + (n-1)(-2)$

$-406 = -2n + 2$

$-408 = -2n$

$n = 204$

$S_n = \frac{n}{2}(a + t_n)$

$= \frac{204}{2}(5 + (-401))$

$= 102(-396)$

$= -40392$

$\therefore S_{204} = -40392$

Ex. 3 Find the sum of the first 42 terms of an arithmetic series with $t_1 = 7$ and $t_{42} = 212$.

$a = 7$

$n = 42$

$t_n = 212$

$S_n = \frac{n}{2}(a + t_n)$

$S_{42} = \frac{42}{2}(7 + 212)$

$= 21(219)$

$= 4599$

Homework

p. 399 #C2, 1adf, 3bc, 4bc, 6-11, 13, 14, 18, 20