6.3 - Geometric Sequences

A sequence where there is a common ratio, r, between consecutive terms. A new term is generated by multiplying/dividing each term by the same number.

$r = \frac{t_2}{t_1}$

eg. $5, 15, 45, 135, \ldots$ ($\times 3$, $\times 3$, $\times 3$) r = 3

$40, 20, 10, 5, 5/2, \ldots$ ($\times \frac{1}{2}$, $\times \frac{1}{2}$) r = $\frac{1}{2}$

$3, -6, 12, -24, 48, \ldots$ ($\times -2$, $\times -2$, $\times -2$) r = -2

Geometric Sequence Formula

$$t_n = ar^{n-1}$$

where a is the first term, and r is the common ratio.

Ex. 1 Find t_7 for each sequence.

a) $t_n = -2(3)^{n-1}$

$n = 7$

$t_7 = -2(3)^{7-1}$

$= -2(3)^6$

$= -1458$

b) $t_n = 100\left(\frac{1}{4}\right)^{n-1}$

$t_7 = 100\left(\frac{1}{4}\right)^6$

$= 100\left(\frac{1}{4096}\right)$

$= \frac{25}{1024}$

Ex. 2 Simplify the powers.

a) $3^{x-1} \bullet 3^{x+5}$

$= 3^{2x+4}$

b) $32^{x+2} \cdot 8^6$ Make the bases the same!

$= (2^5)^{x+2} \cdot (2^3)^6$

$= 2^{5x+10} \cdot 2^{18}$

$= 2^{5x+28}$

Ex. 3 Find t_n for each sequence.

This means find the general formula which works to find any term in the sequence.
Must be simplified.

$t_n = ar^{n-1}$

a) 5, 10, 20, 40, ...

$a = 5$
$r = 2$

$t_n = 5 \cdot 2^{n-1}$

b) 2, 6, 18, 54,.....

$a = 2$
$r = 3$

$t_n = 2 \cdot 3^{n-1}$

c) 6561, 2187, 729, 243, ...

$a = 6561$

$r = \frac{2187}{6561} = \frac{1}{3}$

$t_n = 6561 \cdot \left(\frac{1}{3}\right)^{n-1}$

$= 3^8 \cdot (3^{-1})^{n-1}$

$= 3^8 \cdot 3^{-n+1}$

$= 3^{-n+9}$

d) 3, -12, 48, -192, ...

$a = 3$
$r = -4$

$t_n = 3(-4)^{n-1}$

e) 8, 32, 128, 512,.....

$a = 8$
$r = 4$

$t_n = 8 \cdot 4^{n-1}$

$= 2^3 \cdot 2^{2n-2}$

$= 2^{2n+1}$

f) 1024, -256, 64, -16, ...

$a = 1024$
$r = -\frac{1}{4}$

$t_n = 1024 \cdot \left(-\frac{1}{4}\right)^{n-1}$

$= 4^5 \cdot (-4)^{-n+1}$

$= 4^5 (-1)^{-n+1} \cdot 4^{-n+1}$

$= (-1)^{1-n} \cdot 4^{6-n}$

Ex. 4 Determine the number of terms in each sequence.

a) 5, 20, 80, ... , 81920

$a = 5$
$r = 4$

$t_n = ar^{n-1}$

Sub in 81920 with a & r to solve

$81920 = 5 \cdot 4^{n-1}$
$16384 = 4^{n-1}$
$4^7 = 4^{n-1}$
$\therefore \quad 7 = n - 1$
$n = 8$

$\therefore$ There are 8 terms

b) -19683, 6561, -2187, ... , -3

$a = -19683$
$r = -\frac{1}{3}$

$-3 = -19683\left(-\frac{1}{3}\right)^{n-1}$

$\frac{3}{19683} = \left(-\frac{1}{3}\right)^{n-1}$

$\frac{1}{6561} = \left(-\frac{1}{3}\right)^{n-1}$ or

$\left(-\frac{1}{3}\right)^8 = \left(-\frac{1}{3}\right)^{n-1}$ $\qquad \left(\frac{1}{3}\right)^8 = \ldots$

$\therefore \quad 8 = n - 1$
$9 = n$

$\therefore$ There are 9 terms

Ex. 5 Determine a, r, and t_n for the geometric sequence that has:

a) $t_5 = 324$ and $t_9 = 26244$

$9-5=4$

$324 \xrightarrow{\times r} \square \xrightarrow{\times r} \square \xrightarrow{\times r} \square \xrightarrow{\times r} 26244$

5, 6, 7, 8, 9

$t_n = ar^{n-1}$

$r^4 = \frac{26244}{324}$

$r^4 = 81$

$r = \pm\sqrt[4]{81}$

$\boxed{r = \pm 3}$

Sub $t_n = 324$, $n = 5$

$324 = a(\pm 3)^{5-1}$

$324 = a(\pm 3)^4$

$324 = 81a$

$\boxed{4 = a}$

$$\boxed{t_n = 4(-3)^{n-1} \quad \text{OR} \quad t_n = 4 \cdot 3^{n-1}}$$

b) $t_4 = -8$ and $t_7 = 1$

3

$r^3 = \frac{1}{-8}$

$\boxed{r = -\frac{1}{2}}$

Sub $t_n = 1$, $n = 7$, $r = -\frac{1}{2}$

$1 = a\left(-\frac{1}{2}\right)^{7-1}$

$1 = a\left(-\frac{1}{2}\right)^6$

$1 = a\left(\frac{1}{64}\right)$

$\boxed{64 = a}$

$t_n = 64 \cdot \left(-\frac{1}{2}\right)^{n-1} \rightarrow \left(-2^{-1}\right)^{n-1}$

$= 2^6(-2)^{1-n}$

$= 2^6(-1)^{1-n}(2)^{1-n}$

$= (-1)^{1-n} \cdot 2^{7-n}$

$$\boxed{t_n = (-1)^{1-n} \cdot 2^{7-n}}$$

Ex. 6 Determine the value of x that makes each sequence:

a) geometric

2, 6, 5x - 2

$r = \frac{6}{2}$

$= 3$

$\times 3 = 18$

$5x - 2 = 18$

$5x = 20$

$x = 4$

b) arithmetic

x - 4, 6, x

$d = 6 - (x-4)$ $d = x - 6$

Differences must be the same!

$6-(x-4) = x-6$

$6-x+4 = x-6$

$16 = 2x$

$8 = x$

Be careful of the wording in application problems:

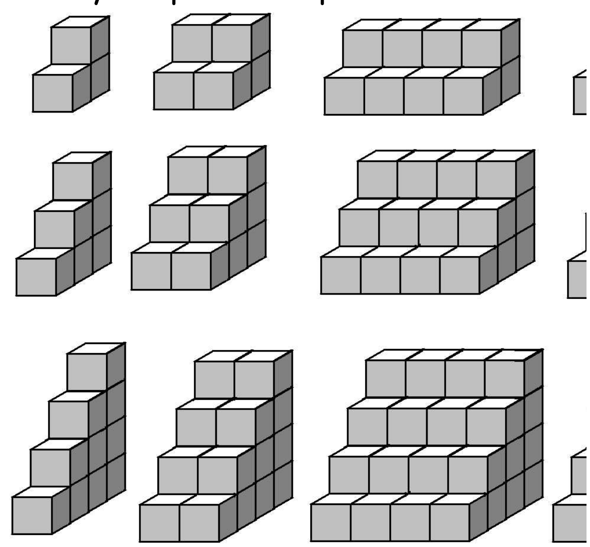
Presently/Now -> t_1

First year -> t_2

HOMEWORK
p. 392 #1, 2bceh, 3ac, 5, 6, 8, 11, 16, 17, 20

or the following geometric sequences, find a fully simplified expression for tn.

a) 729, -243, 81,....
b) t4=64 and t5=32
c) t2=4 and t4=64



Answers:
a) tn=(-1)^(n-1)(3)^(7-n)
b) tn=2^(10-n)
c) tn=4^(n-1) OR tn=(-1)^n(4)^(n-1)