6.2 - Arithmetic Sequences

A sequence where there is a <u>common difference</u>, d, between consecutive terms. The same value is added or subtracted to a term to generate the next term.

eg. 3, 5, 7, 9, 11, ...   d = 2

5, 1, -3, -7, ...   d = -4

0, 5, 10, 15, 20,..   d = 5

Notice the pattern: 1, →+3 4, →+3 7,   10,   13

1, 1 + 1(3), 1 + 2(3), 1 + 3(3), 1+4(3)   a = 1   d = 3

a,   a + 1d,   a + 2d,   a + 3d,   a + 4d

$t_n = a + (n-1)d$

always 1 less than term #

Arithmetic Sequence Formula

$t_n = a + (n - 1)d$

where a is the first term and d is the common difference

$a = t_1$

Ex. 1 Determine t_n for each.

> *This means find the general formula which works to find any term in the sequence.*
> *Must be simplified.*

a) 7, 3, -1, -5, ...

$a = 7$
$d = -4$

$$t_n = a + (n-1)d$$
$$= 7 + (n-1)(-4)$$
$$= 7 - 4n + 4$$
$$\boxed{t_n = 11 - 4n}$$

b) -5, -3, -1, 1, ...

$a = -5$
$d = 2$

$$t_n = a + (n-1)d$$
$$= -5 + (n-1)(2)$$
$$= -5 + 2n - 2$$
$$\boxed{t_n = 2n - 7}$$

Ex. 2 Determine the # of terms in each sequence.

a) 2, 5, 8, ..., 155

$a = 2$
$d = 3$

① $t_n = a + (n-1)d$
$$= 2 + (n-1)(3)$$
$$\boxed{t_n = 3n - 1}$$

②
$$155 = 3n - 1$$
$$156 = 3n$$
$$n = 52$$
$$\therefore 52 \text{ terms}$$

b) 1, -1, -3, ..., -199

$a = 1$
$d = -2$

① $t_n = a + (n-1)(-2)$
$$= 1 - 2n + 2$$
$$\boxed{t_n = 3 - 2n}$$

②
$$-199 = 3 - 2n$$
$$-202 = -2n$$
$$101 = n$$
$$\therefore 101 \text{ terms}$$

Ex. 3 Insert two numbers between 17 and 59, so that the four numbers form an arithmetic sequence.
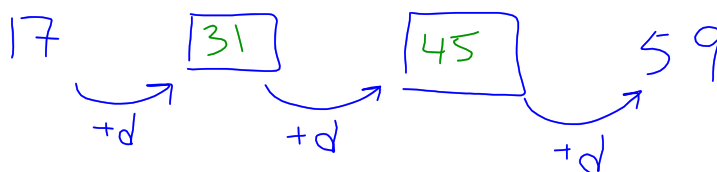
17    $\boxed{31}$ $\boxed{45}$ 59

+d   +d   +d

$$17 + 3d = 59$$
$$3d = 42$$
$$d = 14$$

$$t_1 = 17$$
$$t_2 = 17 + 14$$
$$= 31$$

$$t_3 = 31 + 14$$
$$= 45$$

4. Determine, a, d, and t_n for each arithmetic sequence.

a) $t_4 = 13$, $t_{17} = 39$

13 terms
(17 - 4)

t_4
$13 + 13d = 39$
$13d = 26$
$\boxed{d = 2}$

$t_n = a + (n-1)d$
Using $t_4 = 13$

$13 = a + (4-1)(2)$
$13 = a + 6$
$\boxed{7 = a}$

$t_n = a + (n-1)d$
$\quad = 7 + (n-1)(2)$
$\boxed{t_n = 2n + 5}$

b) $t_{10} = -67$, $t_{43} = -298$

$33d$

$-67 + 33d = -298$
$33d = -231$
$\boxed{d = -7}$

$t_n = a + (n-1)d$ $\begin{cases} \text{Using } t_{43} \\ n = 43 \\ t_{43} = -298 \end{cases}$

$-298 = a + (43-1)(-7)$
$-298 = a - 301 + 7$
$-298 = a - 294$
$\boxed{a = -4}$

$t_n = a + (n-1)d$

$t_n = -4 + (n-1)(-7)$
$\quad = -4 - 7n + 7$
$\boxed{t_n = 3 - 7n}$

p. 385 #3abfh, 4bc, 6, 7, 9ac, 10ac, 11ac, 13, 15, 20, 21