6.1 - Sequences and Recursive Procedures

Sequence: A set of numbers arranged in a particular order.

Term: Each number (or letter) in a sequence is referred to as a term.
We use t_1, t_2, etc. to denote term numbers.

$t_1 = 3$ → 3,5,7,9 ***finite sequence***

-1, -4, -7, -10, ... ***infinite sequence***

In some cases, the pattern of a sequence can be defined using a general rule (equation). This rule is dependent on the term #, n, and describes how to generate t_n (the n^{th} term).

Note: $n \geq 1$ since values of n are natural numbers {1,2,3,4...}

Ex.1 Find the first 3 terms of each sequence.

a) $t_n = 3n - 2$

$t_1 = 3(1) - 2$
$= 1$

$t_2 = 3(2) - 2$
$= 4$

$t_3 = 3(3) - 2$
$= 7$

b) $t_n = n^2 - 3$

$t_1 = (1)^2 - 3$
$= -2$

$t_2 = (2)^2 - 3$
$= 1$

$t_3 = (3)^2 - 3$
$= 6$

These formulas are called explicit formulas. They can be used to find any term by using the term #, n.

$\boxed{t_n \text{ can also be written using function notation: } f(n)}$

Ex. 2 Given f(n), determine t_{10}.

a) $f(n) = 2n^2 - n$

$f(10) = 2(10)^2 - 10$
$= 200 - 10$
$= 190$

b) $f(n) = 5 - n$

$f(10) = 5 - 10$
$= -5$

Sequences can be represented graphically. The independent variable is n, the term #, and the dependent variable is t_n or f(n), the value of the term.

Note: Since values of n are Natural Numbers {1,2,3,4...}, the graphs are always a series of points, rather than a line or curve.

Ex. 3 Graph the first 3 terms of each sequence.

a) $t_n = 2n + 1$

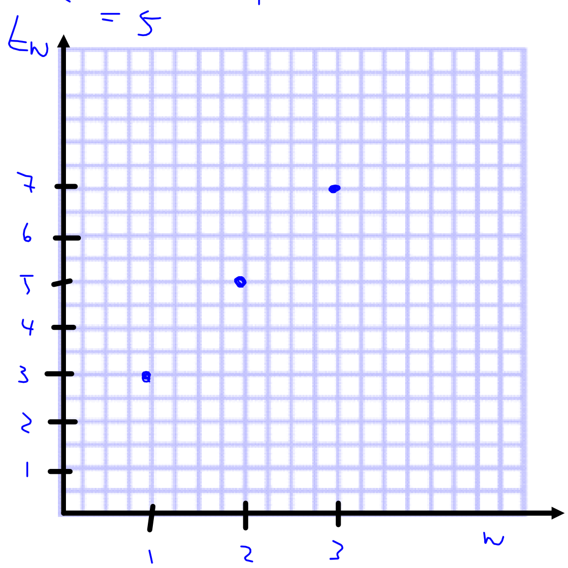
$t_1 = 2(1) + 1$
$= 3$

$t_2 = 2(2) + 1$
$= 5$

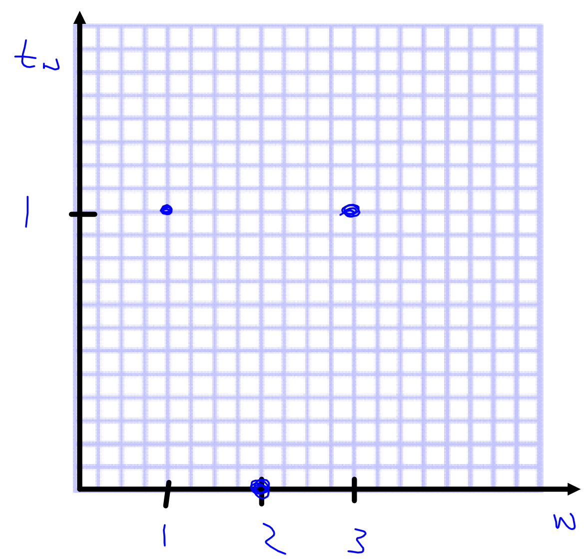
$t_3 = 2(3) + 1$
$= 7$

b) $f(n) = (n - 2)^2$

$f(1) = 1$
$f(2) = 0$
$f(3) = 1$



LINEAR



QUADRATIC

Ex. 4 Determine a formula for each sequence.

a) 3, 5, 7, 9,... t_1 t_2 t_3 t_4 +2 +2 $t_n = 2n+1$

b) 4, 8, 12, 16,... +4 +4 $t_n = 4n$

c) 2, 6, 10, 14,... +4 $t_n = 4n - 2$

Check!

n	t_n
1	2
2	6
3	10

✓

d) 4, 1, -2, -5,... −3 −3 −3 $t_n = -3n + 7$

e) 2, 4, 8, 16,... ×2 ×2 ×2 $t_n = 2^n$

$$\left\{\begin{aligned} t_1 &= 2 \\ t_2 &= 2 \cdot 2 \\ t_3 &= 2 \cdot 2 \cdot 2 \end{aligned}\right.$$

f) 5, 25, 125, 625,... $t_n = 5^n$

g) 2, 6, 18, 54, 162,... ×3 ×3 ×3 $t_n = 2 \cdot 3^{n-1}$

$$\left\{\begin{aligned} t_1 &= 2 \\ t_2 &= 2 \cdot 3 \\ t_3 &= 2 \cdot 3 \cdot 3 \end{aligned}\right.$$

$$t_n = 2 \cdot 3^{n-1}$$

$$\left\{\begin{aligned} &2 \cdot 3^0 \\ &2 \cdot 3^1 \\ &2 \cdot 3^2 \end{aligned}\right.$$

h) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ $t_n = \frac{1}{n+1}$

i) -1, 2, -3, 4, -5,... $t_n = (-1)^n \cdot n$

Recursion formulas are another way to describe the terms of a sequence. They require the use of previous terms to generate new terms.

Ex. 5 Use the given recursion formula to generate the first 4 terms.

a) $t_1 = 4$, $t_n = 3 - 4t_{n-1}$

$t_2 = 3 - 4t_1$
$= 3 - 4(4)$
$= 3 - 16$
$= -13$

$t_3 = 3 - 4t_2$
$= 3 - 4(-13)$
$= 55$

$t_4 = 3 - 4(55)$
$= -217$

b) $t_1 = 2$, $t_2 = 3$, $t_n = 2t_{n-2} - 3t_{n-1}$

$t_3 = 2t_1 - 3t_2$
$= 2(2) - 3(3)$
$= 4 - 9$
$= -5$

$t_4 = 2t_2 - 3t_3$
$= 2(3) - 3(-5)$
$= 6 + 15$
$= 21$

c) $f(1) = 3$, $f(2) = -1$, $f(n) = f(n-1) + 2f(n-2)$

These sequences can also be written in function notation.

$f(3) = f(3-1) + 2f(3-2)$
$= f(2) + 2f(1)$
$= -1 + 2(3)$
$= 5$

$f(4) = f(4-1) + 2f(4-2)$
$= f(3) + 2f(2)$
$= 5 + 2(-1)$
$= 3$

Ex.6 Write a recursion formula for the Fibonacci Sequence.

1, 1, 2, 3, 5, 8, 13, 21, ...

TEDTALK

$t_n = t_{n-1} + t_{n-2}$, $n \geq 3$

where $t_1 = 1$

$t_2 = 1$

Ex.7 Write an explicit formula for each recursion formula.

a) $t_1 = 3$, $t_n = t_{n-1} + 4$

$t_2 = 3 + 4$
$= 7$

$t_3 = 7 + 4$
$= 11$

$t_4 = 11 + 4$
$= 15$

3, 7, 11, 15, ...

$\boxed{t_n = 4n - 1}$

b) $t_1 = -2$, $t_n = 3t_{n-1}$

$t_2 = 3(-2)$
$= -6$

$t_3 = 3(-6)$
$= -18$

$t_4 = 3(-18)$
$= -54$

-2, -6, -18, -54

-2

$-2 \cdot 3$

$-2 \cdot 3 \cdot 3$

$-2 \cdot 3 \cdot 3 \cdot 3$

Maybe

$-2 \cdot 3^{n-1}$?

$\boxed{t_n = -2 \cdot 3^{n-1}}$

HOMEWORK
p.360 # 1adef, 3, 4, 8, 13
p.370 # 1abde, 2ac, 3, 8af, 9ab, 16