5.6B Applications of Trig Functions-continued

Ex 1. The temperature of a solar-heated pool changes throughout a sunny day and is modeled by a trigonometric relation. The temperature ranges from 23°C at 6 A.M. to 29°C at 6 P.M.

MIN MAX

MIN → MAX
$\frac{1}{2}$ period

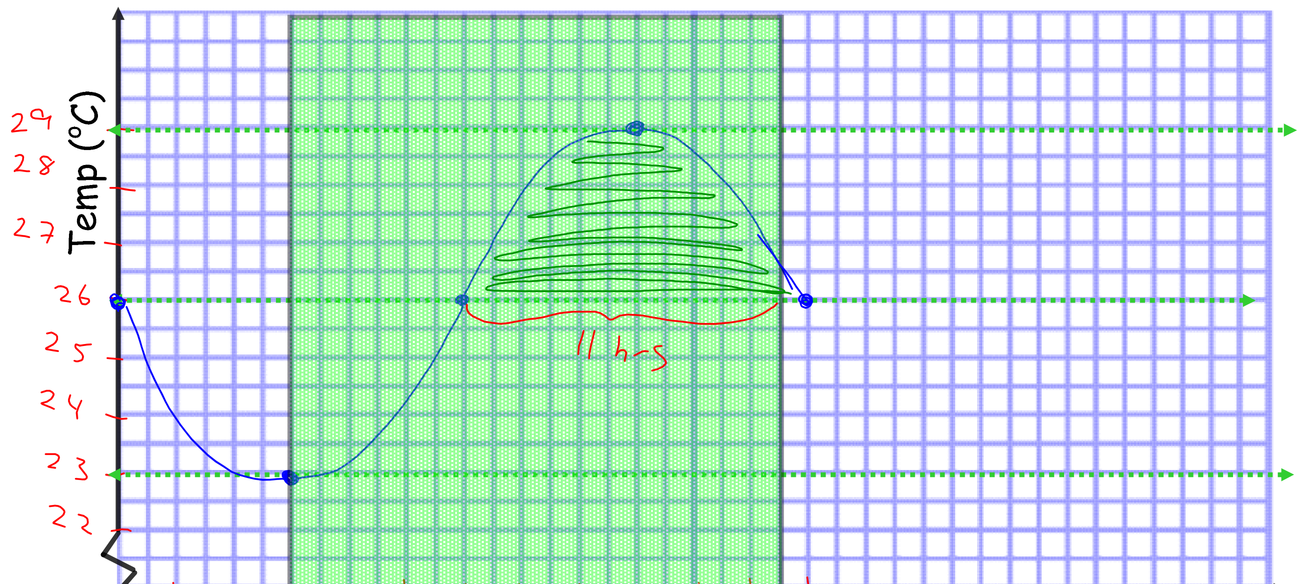
∴ Period = 24 hrs

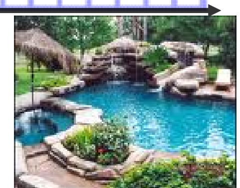
a) Graph the relation for a 24 hour period starting at midnight (t=0).

$c = \frac{max+min}{2}$
$= 26$

$a = \frac{max-min}{2}$
$= 3$

spacing $= \frac{24}{4}$
$= 6$ hrs





b) Determine the equation of a sine function for the given graph.

$k = \frac{360}{24}$
$= 15$

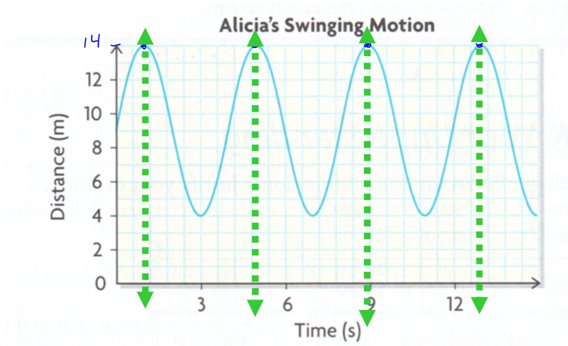
$y = -3\sin(15t) + 26$ OR $y = 3\sin[15(t-12)] + 26$

c) The pool is comfortable for swimming when the temperature is at least 26°C. The pool is open from 6 A.M. to 11 P.M. every day. How many hours of comfortable swimming are available on a sunny day?

(From graph)

11 hrs each day

Ex 2: Alicia was swinging back and forth in front of a motion detector. Her distance from the detector can be modelled by the equation $d(t)=5\sin 90^\circ t + 9$.



a) Find the times when Alicia is 14 m away for the number of swings given. Show graphically and algebraically.

Graphically

1s, 5s, 9s, 13s

Approx.

Algebraically

$d(t)=5\sin(90t)+9$

Sub $d=14$

$14=5\sin(90t)+9$

$14-9=5\sin(90t)$

$\frac{5}{5}=\sin(90t)$

$1=\sin(90t)$

$\sin^{-1}(1)=90^\circ t$

$90^\circ=90^\circ t$

$1=t$

Period $=\frac{360}{90}$

$=4s$

$t_1=1s$

$t_2=1s+4s$

$t_3=1s+4s+4s$

$\therefore$ Alicia is 14m away at $t=1, 5, 9$, etc

b) Find the times when Alicia is 12 m away algebraically for the number of swings given. Round to one place.

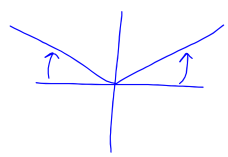
$d(t)=5\sin(90^\circ t)+9$

Sub $d=12$

$12=5\sin(90^\circ t)+9$

$\frac{3}{5}=\sin(90t)$

$\sin^{-1}\left(\frac{3}{5}\right)=90t$



Q1

$36.9^\circ=90^\circ t_1$

$t_1=\frac{36.9}{90}$

$\doteq 0.4$

Q2

$180-36.9=90^\circ t_2$

$143.1=90^\circ t_2$

$\frac{143.1}{90}=t_2$

$t_2\doteq 1.6$

$t_3=0.4+4$

$=4.4$

$t_4=1.6+4$

$=5.6$

$t_5=0.4+8$

$=8.4$

$t_6=1.6+8$

$=9.6$

$t_7=0.4+12$

$=12.4$

$t_8=1.6+12$

$=13.6$

$\therefore$ Alicia is 12m away at

0.4, 1.6, 4.4, 5.6, 8.4, 9.6, 12.4, 13.6

Ex 3: The depth of the water in a harbour fluctuates because of the tide anc is modeled by the equation d = -3cos(30t) + 6 ,
where d represents the depth of the water in metres, and t represents the number of hours after midnight.
(ie. t=0 means midnight, t=3 means 3 A.M. etc)

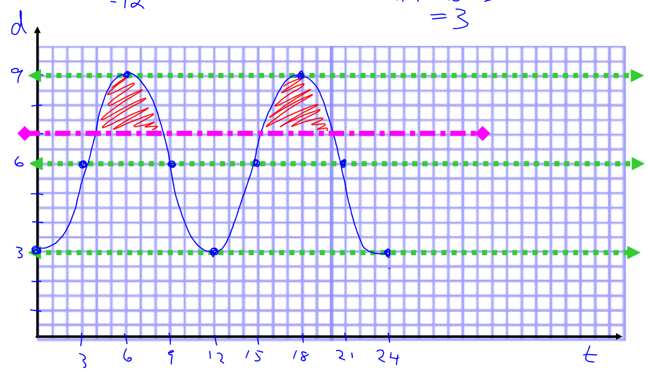
a) Graph the relation for 24 hours

Period $= \frac{360}{30}$
$= 12$

spacing $= \frac{12}{4}$
$= 3$

Max $= 6 + 3$
$= 9$

Min $= 6 - 3$
$= 3$



b) Determine the value of d when t=3. Explain what these values represent.

With graph:

6m

∴ At 3s, depth is 6m

With equation:

$d = -3\cos(30t) + 6$
Sub $t = 3$
$d = -3\cos(30 \cdot 3) + 6$
$= 6$

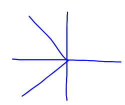
c) Determine the maximum depth of the water

9m

d) Surfing is allowed when the depth of the water is 7 metres or more. Show graphically and <u>algebraically</u> when this occurs?

Set d=7 and solve

$d = -3\cos(30t) + 6$
$7 = -3\cos(30t) + 6$
$-\frac{1}{3} = \cos(30t)$
$\cos^{-1}\left(-\frac{1}{3}\right) = 30t$

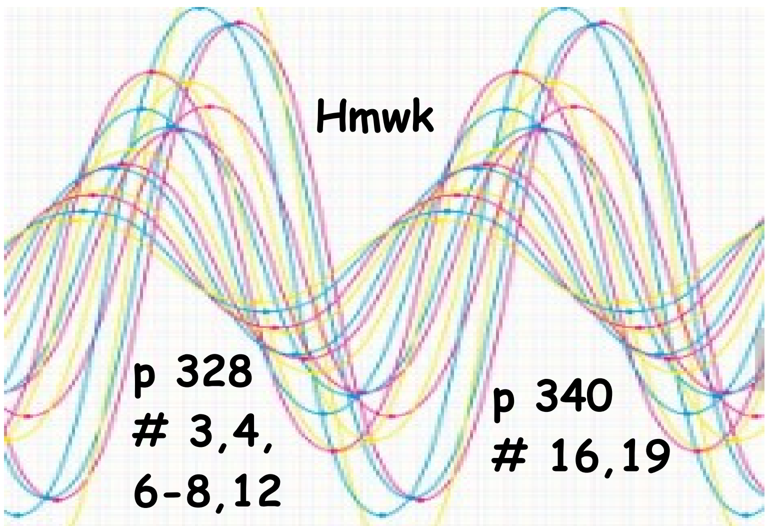


$\theta_r = 180 - 109.5$
$= 70.5°$

Q2
$109.5° = 30t$
$\frac{109.5}{30} = t$
$3.6 = t$

Q3
$180 + 70.5 = 30t$
$\frac{250.5}{30} = t$
$8.35 \doteq t$

How long above?
$t = 8.35 - 3.6$
$= 4.7$ hrs


Hmwk
p 328
3,4,
6-8,12
p 340
16,19