5.4 -Combination of Transformations

Vertical Stretch/Compression (AMPLITUDE)

Horizontal Shift left/right (PHASE SHIFT)

In general: $y = af(k(x-d))+c$

Horizontal Stretch/Compression

Vertical Shift up/down

Function axis "Imaginary Middle"

Perform transformations in the following order:

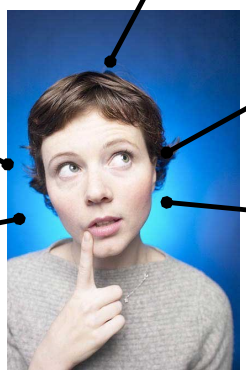
1) Reflections and Stretches/compressions together
2) Translations

Don't forget to factor out the coefficient of the x-term!

period= $\frac{360^0}{k}$

The amplitude is |a|

$$a = \frac{\max - \min}{2}$$



Max= c + |a|
Min = c - |a|

y=c is the base line/imaginary middle (axis of the curve)

$$y = \frac{\max + \min}{2}$$

Ex 1 - Describe the transformations from the base function.

a) $y = -2\sin 3x$

① Reflection in x-axis
② V.S. by 2
③ H.C. by 3

b) $h(x) = 3\sin(x - 60^0) + 2$

① V.S. by 3
② Phase shift 60° right
③ Vert. shift up 2

c) $y = 2\sin(2x + 45^0) + 3$

$2(x+22.5)$

① V.S. by 2
② H.C. by 2
③ Phase shift 22.5° left
④ Vert. shift up 3

d) $y = \frac{-1}{2}\cos(4x-180^0)$

$4(x-45^0)$

① Reflection in x-axis
② Vertical Compression by 2
③ H.C. by 4
④ Phase shift 45° right

Ex 2 - Sketch the graph for each, pay attention to the restrictions. State the amplitude, period, D & R, phase shift and vertical translation.

a) $y = 3\sin\left(\frac{1}{2}x - 30°\right) + 0$ for one cycle

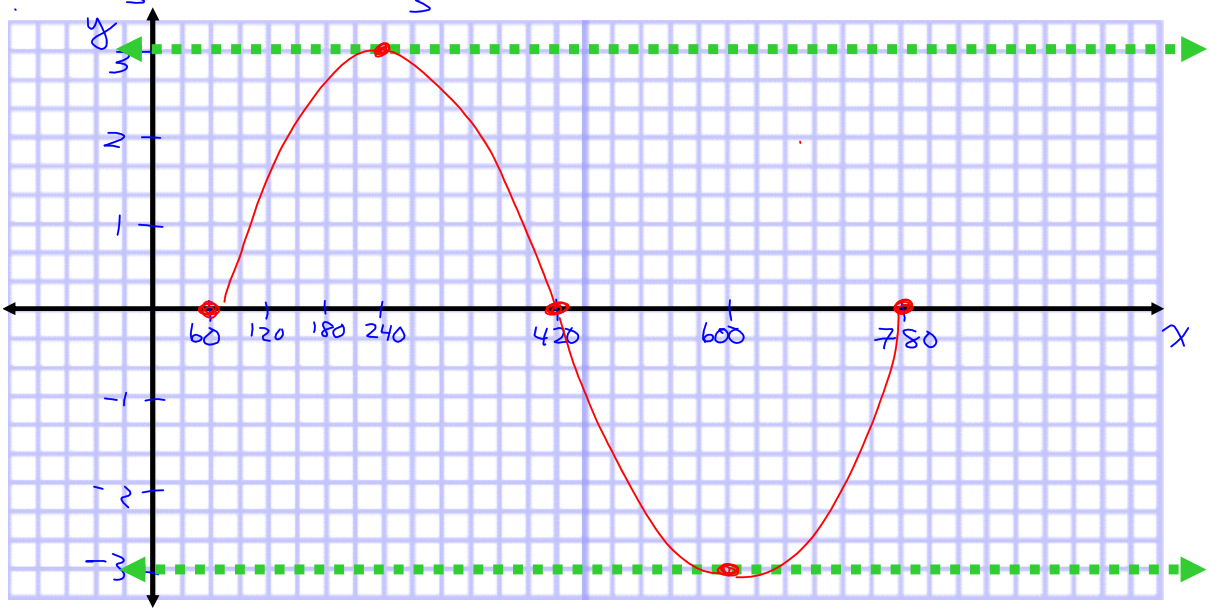
$y = 3\sin\left[\frac{1}{2}(x - 60°)\right]$

D: $\{x \in \mathbb{R} \mid 60° \le x \le 780°\}$

R: $\{y \in \mathbb{R} \mid -3 \le y \le 3\}$

period $= \frac{360}{\frac{1}{2}} = 720$ spacing $= \frac{720}{4} = 180$

max $= 0 + 3 = 3$ min $= 0 - 3 = -3$



b) $y = -2\cos(2x - 90^0) - 2$, $0 \le x \le 360^o$

$y = -2\cos[2(x - 45^o)] - 2$

$D: \{x \in \mathbb{R} \mid 0 \le x \le 360^o\}$

$R: \{y \in \mathbb{R} \mid -4 \le y \le 0\}$

$\text{period} = \frac{360}{2}$

$= 180$

$\text{spacing} = \frac{180}{4}$

$= 45^o$

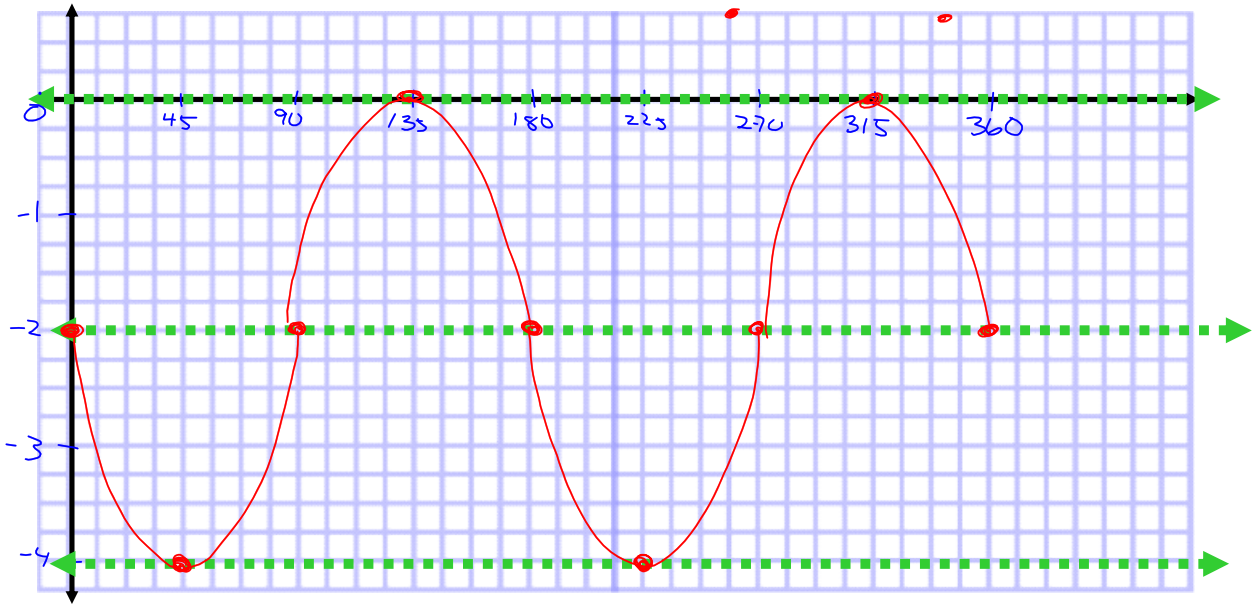
$\text{max} = -2 + 2$

$= 0$

$\text{min} = -2 - 2$

$= -4$

$-\cos$



c) $y = \frac{1}{2}\cos(3x - 90°) + 1$, $0 \le x \le 360^{\circ}$

$y = \frac{1}{2}\cos[3(x - 30°)] + 1$

period $= \frac{360}{3}$
$= 120$

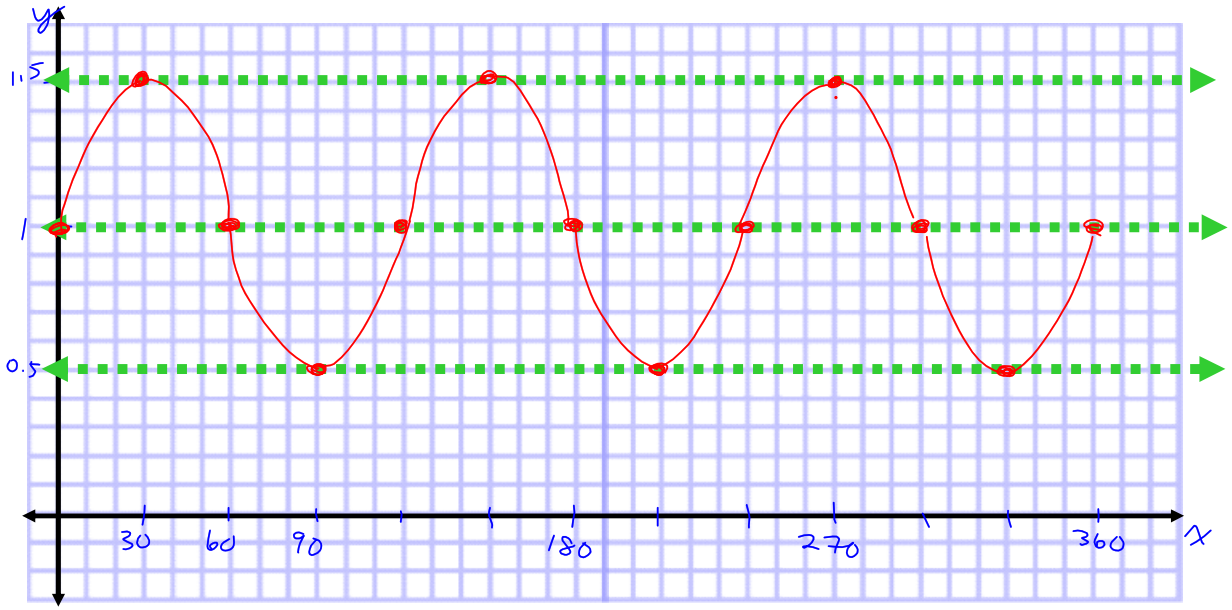
spacing $= \frac{120}{4}$
$= 30°$

max $= 1 + 0.5$
$= 1.5$

min $= 1 - 0.5$
$= 0.5$

$D: \{x \in \mathbb{R} \mid 0° \le x \le 360°\}$

$R: \{y \in \mathbb{R} \mid 0.5 \le y \le 1.5\}$



Homework
p 310 # 10, 11a (and graph by hand i, ii, iii)
p318 # 1, 2
5.4 Handout