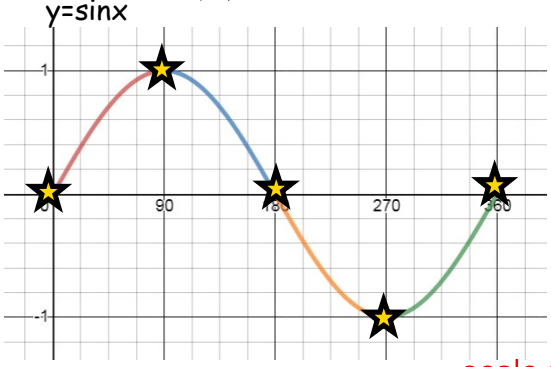
5.3 Stretches of Periodic Functions

To sketch sine and cosine functions, remember the 5 key points: Maximum, Minimum, and zeroes. These 5 points are equally spaced along the x-axis, so they divide the period into quarters.

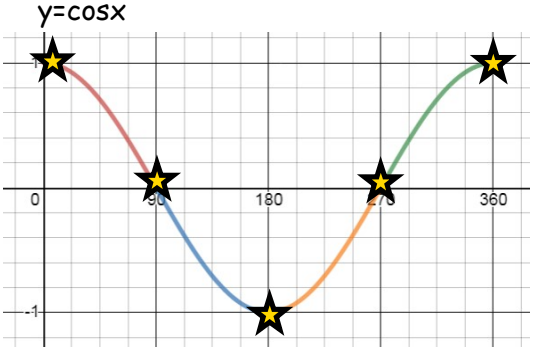
scale = period / 4

Key points

y=sinx



y=cosx



scale = 360^o / 4
= 90^o
(something happens every 90^o)

Vertical Compressions and stretches

Given $y = af(x)$, when :

$|a| > 1$ vertical stretch by a

$0 < |a| < 1$ vertical compression by $\frac{1}{a}$

$a < 0$ reflection in x-axis

"a" is called the amplitude

ex: $y = 2\sin x$

↑ Vertical stretch by 2

ex $y = \frac{1}{3}\cos x$

↑ Vertical compression by 3

Ex 1 Sketch the following for one full cycle.

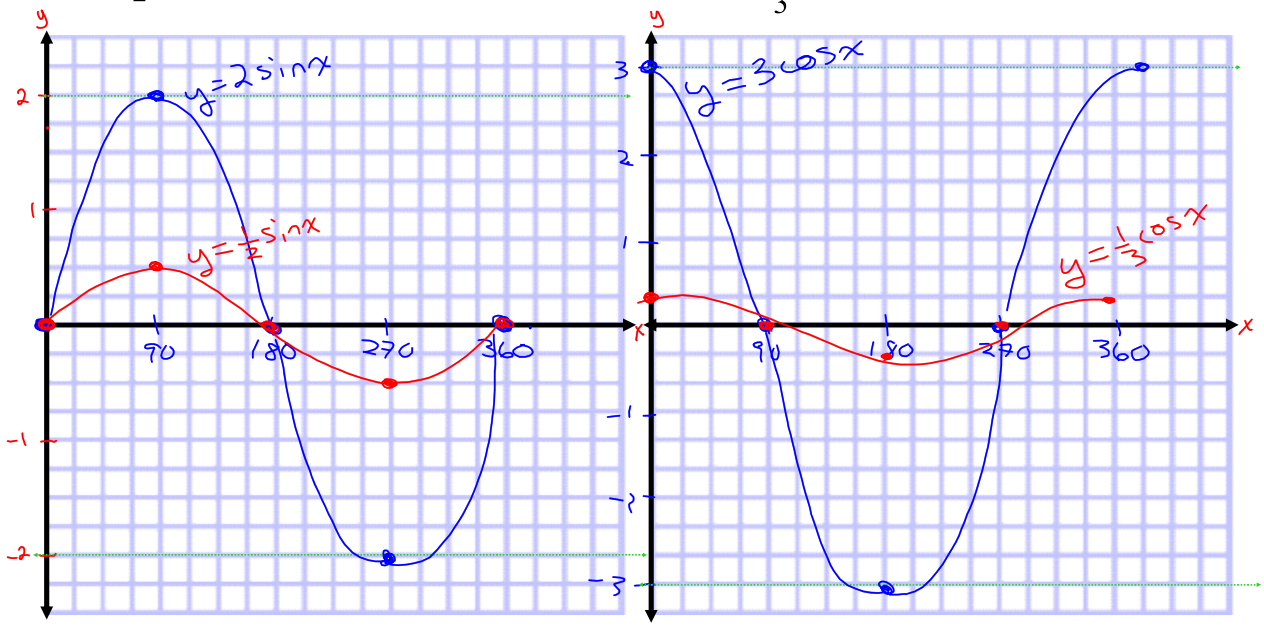
Base $y = \sin x$

a) $y = 2\sin x$

b) $y = \frac{1}{2}\sin x$

c) $y = 3\cos x$

d) $y = \frac{1}{3}\cos x$



For the above examples:

Does the amplitude change? YES

Does the period change? NO

Horizontal Stretches or Compressions

Remember, k is INSIDE the function and behaves OPPOSITE from what you would expect.

Given y = f(kx), when:

$|k| > 1$ Horizontal compression by k

$0 < |k| < 1$ Horz. stretch by $\frac{1}{k}$

$k < 0$ Horz. Reflection (in y-axis)

Because you are stretching/compressing horizontally, the period would change.

$$\text{PERIOD} = \frac{360^0}{k}$$

Recall:

$$\text{Scale} = \frac{\text{period}}{4}$$

Ex 2 Sketch the following for one full cycle.

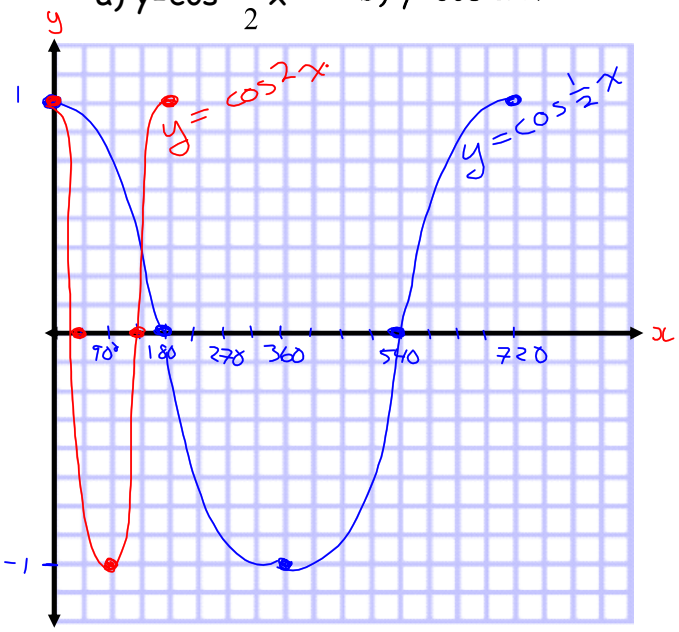
a) $y=\cos\frac{1}{2}x$ b) $y=\cos 2x$

a) period $= \frac{360}{k}$
$= \frac{360}{\frac{1}{2}}$
$= 720$

spacing $= \frac{720}{4}$
$= 180°$

b) period
$= \frac{360}{2}$
$= 180$

spacing $= \frac{180}{4}$
$= 45°$



For these examples:
Does the amplitude change?
Does the period change?

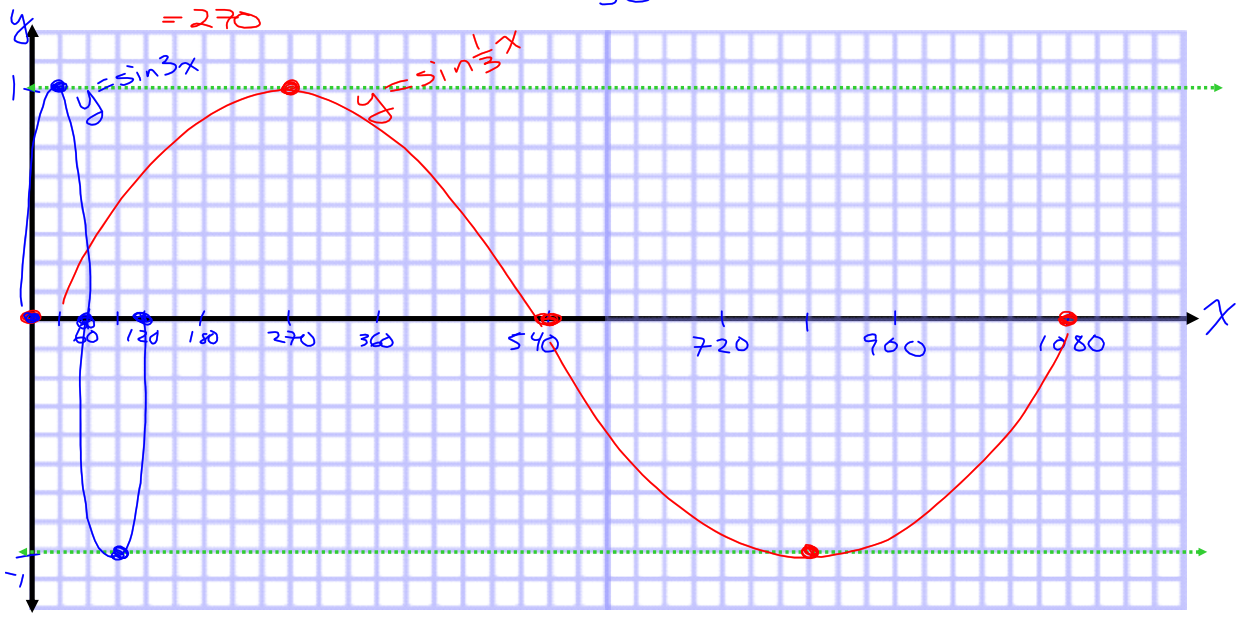
c) $y=\sin\frac{1}{3}x$ d) $y=\sin 3x$

period $= \frac{360}{\frac{1}{3}}$
$= 1080$

spacing $= \frac{1080}{4}$
$= 270$

period $= \frac{360}{3}$
$= 120$

spacing $= \frac{120}{4}$
$= 30$



Ex 3 Graph one cycle of $y = 2\sin\left(\frac{1}{2}x\right)$

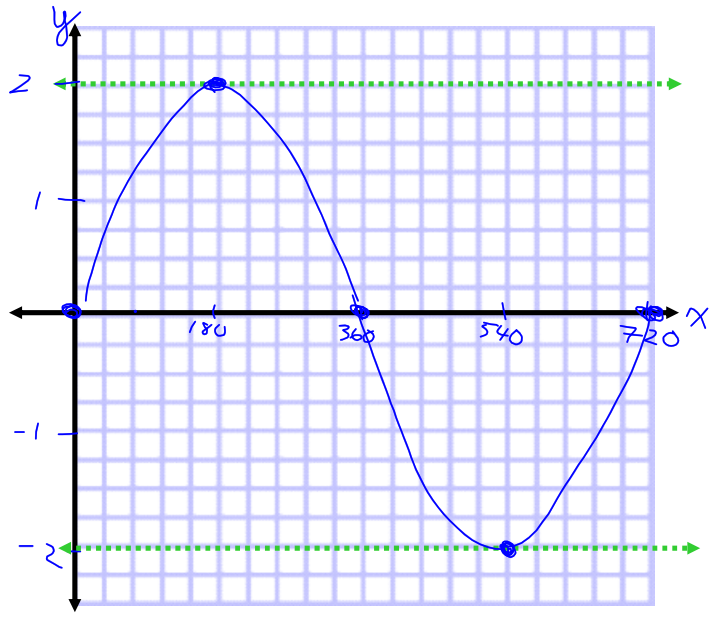
period $= \frac{360}{\frac{1}{2}}$

$= 720$

spacing $= \frac{720}{4}$

$= 180$

amplitude $= 2$



Homework - p. 309 # 1ac, 2bd, 3 (and sketch a,c,f,h), p 312 #18