3.6 - Solving Exponential Equations

$$5^x = 5^4$$

then $x = 4$

If two powers are equal and bases are same then the exponents must be equal

Ex. 1 Solve.

For an equation to be true, both sides must be equal.
If $a^x = a^y$, then x = y except when a = -1, 0 or 1.
∴ Convert powers to a common base and solve by comparing exponents.

a) $2^x = 32$

$2^x = 2^5$

then $x = 5$

b) $(-4)^x = -1024$ ← notice here base is −4

$(-4)^x = (-4)^5$

then $x = 5$

c) $-3^y = -81$ ← no brackets base is 3

Isolate power

$3^y = 81$

$3^y = 3^4$

then $y = 4$

d) $5^{4-3x} = 25^{x+1}$

$5^{4-3x} = (5^2)^{x+1}$ power of power mult exp.

then

$4 - 3x = 2(x+1)$

$4 - 3x = 2x + 2$

$2 = 5x$

$\frac{2}{5} = x$

e) $3^{x+1} = 1$ ← * need like bases

$3^0 = 1$

$3^{x+1} = 3^0$

then

$x + 1 = 0$

$x = -1$

f) $4^{x+5} = 8^{1-3x}$

$(2^2)^{x+5} = (2^3)^{1-3x}$

then

$2(x+5) = 3(1-3x)$

$2x+10 = 3-9x$

$11x = -7$

$x = -\frac{7}{11}$

g) $7^{3-x} = \frac{1}{49}$ invert base use neg exp

$7^{3-x} = 7^{-2}$

then

$3-x = -2$

$5 = x$

h) $32(2^{x+5}) - 6 = -4$ isolate pow

$32(2^{x+5}) = 2$

$2^{x+5} = \frac{2}{32}$ $\frac{1}{16}$

$2^{x+5} = 2^{-4}$

then

$x+5 = -4$

$x = -9$

Practice:

i) $\frac{1}{256} = 4^{5x+1}$

$4^{-4} = 4^{5x+1}$

then

$-4 = 5x+1$

$-5 = 5x$

$-1 = x$

j) $9^{2x+3} = 27^{\frac{x}{4}}$

$(3^2)^{2x+3} = (3^3)^{x/4}$

then

$4 \cdot 2(2x+3) = 3(\frac{x}{4}) \cdot 4$

$8(2x+3) = 3x$

$16x+24 = 3x$

$13x = -24$

$x = -\frac{24}{13}$

k) $4(3^{5x-1}) = 36$

$3^{5x-1} = 9$

$3^{5x-1} = 3^2$

then

$5x-1 = 2$

$5x = 3$

$x = \frac{3}{5}$

But what if we can't make the bases the same?

Not a problem this year! Next year, you will learn logarithms.

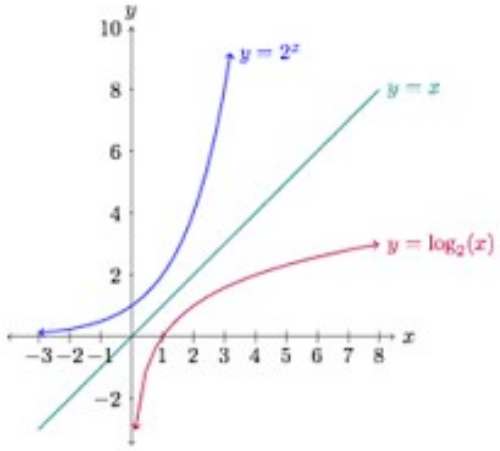
$$4^{x+2} = 14$$

$$\log 4^{x+2} = \log 14$$

$$(x+2)\log 4 = \log 14$$

$$x + 2 = \frac{\log 14}{\log 4}$$

$$x = \frac{\log 14}{\log 4} - 2 \approx -0.96322$$



Ex. 2 Solve.

a) $2^{x+4} + 2^x = 136$

The x-values must be 'brought together' in order to solve.
This can be accomplished by dividing out a common factor.

$2^x(2^4)+2^x = 136$

GF 2^x

$2^x(2^4+1) = 136$

$2^x(16+1) = 136$

$2^x(17) = 136$

$2^x = 8$

$2^x = 2^3$

then $x = 3$

Work exp laws backwards

$(2^x)(2^4)$ mult power if same base add exp

2^{x+4}

b) $3^{x+1} - 3^{x+4} = -702$

$3^x(3-3^4) = -702$

$3^x(-78) = -702$

$3^x = 9$

$3^x = 3^2$

then $x = 2$

Practice:

c) $7^{x+1} + 7^{x+2} = 392$

$7^x(7+7^2) = 392$

$7^x(56) = 392$

$7^x = 7$

then $x = 1$

method 2 GCF

$3^{x+1} - 3^{x+4} = -702$

$3^{x+1}(1-3^3) = -702$

$3^{x+1}(-26) = -702$

$3^{x+1} = 27$

$3^{x+1} = 3^3$

then

$x+1 = 3$

$x = 2$

Homework

#1 - 6 (Pick and choose)