

3.3 - Rational Exponents

when the exponent is a rational number.

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

index

index: indicates what root you want

Evaluate

$$8^{\frac{1}{3}}$$

$$= \sqrt[3]{8}$$

$$= 2$$

$$9^{\frac{1}{2}}$$

$$= \sqrt{9}$$

$$= 3$$

👉 Note: when the index is 2 we don't write it... it is understood (square root)

Ex 1. Evaluate.

a) $27^{\frac{1}{3}}$

$$= \sqrt[3]{27}$$

$$= 3$$

b) $(-8)^{\frac{1}{3}}$

$$= \sqrt[3]{-8}$$

$$= -2$$

c) $(-16)^{\frac{1}{2}}$

$$= \sqrt{-16}$$

∴ No real roots!

d) $-16^{\frac{1}{2}}$

$$= -\sqrt{16}$$

$$= -4$$

e) $16^{\frac{-1}{4}}$

$$= \frac{1}{16^{\frac{1}{4}}}$$

$$= \frac{1}{\sqrt[4]{16}}$$

$$= \frac{1}{2}$$

f) $64^{\frac{-1}{2}}$

$$= \frac{1}{64^{\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{64}}$$

$$= \frac{1}{8}$$

i) $81^{\frac{-1}{4}}$

$$= \frac{1}{\sqrt[4]{81}}$$

$$= \frac{1}{3}$$

numerator is the exponent

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

OR

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

denominator is the index



Ex 2. Evaluate.

a) $27^{\frac{2}{3}}$
 $= (\sqrt[3]{27})^2$
 $= 3^2$
 $= 9$


b) $81^{\frac{5}{4}}$
 $= (\sqrt[4]{81})^5$
 $= 3^5$
 $= 243$

c) $16^{\frac{-3}{2}}$
 $= \frac{1}{16^{\frac{3}{2}}}$
 $= \frac{1}{(\sqrt{16})^3}$
 $= \frac{1}{64}$


d) $-8^{\frac{2}{3}}$
 $= -(\sqrt[3]{8})^2$
 $= -2^2$
 $= -4$

e) $(-27)^{\frac{-1}{3}}$
 $= \frac{1}{\sqrt[3]{-27}}$
 $= \frac{1}{-3}$
 $= -\frac{1}{3}$

f) $\left(\frac{4}{81}\right)^{\frac{-3}{2}}$
 $= \left(\frac{81}{4}\right)^{\frac{3}{2}}$
 $= \frac{(\sqrt{81})^3}{(\sqrt{4})^3}$
 $= \frac{729}{8}$

g) $32^{0.4}$ 
 $= 32^{\frac{4}{10}}$
 $= 32^{\frac{2}{5}}$
 $= (\sqrt[5]{32})^2$
 $= 2^2$
 $= 4$

Hint:
Change 0.4 into a
fraction in lowest
terms

h) $25^{\frac{4}{9}} \cdot 5^{\frac{1}{9}}$ 
 $= (5^2)^{\frac{4}{9}} \cdot 5^{\frac{1}{9}}$
 $= 5^{\frac{8}{9}} \cdot 5^{\frac{1}{9}}$
 $= 5$

Hint:
change to
like bases

Ex 3. Write the following radicals in exponent form.

ALWAYS REPLACE RADICAL w/ BRACKETS



a) $\sqrt[5]{7x^4}$
 $= (7x^4)^{\frac{1}{5}}$
 $= 7^{\frac{1}{5}} x^{\frac{4}{5}}$

b) $\sqrt[3]{\sqrt{x^5}}$
 $= ((x^5)^{\frac{1}{2}})^{\frac{1}{3}}$
 $= x^{\frac{5}{6}}$

c) $\sqrt[3]{-5x^4}$
 $= (-5x^4)^{\frac{1}{3}}$
 $= (-5)^{\frac{1}{3}} x^{\frac{4}{3}}$

d) $\frac{1}{\sqrt[7]{x^9}}$
 $= \frac{1}{(x^9)^{\frac{1}{7}}}$
 $= \frac{1}{x^{\frac{9}{7}}}$

$x^{-\frac{9}{7}}$
 BAD
 (C)

f) $(\sqrt[3]{x^3 y^2})(\sqrt[4]{x^{-2} y^3})$
 $= (x^3 y^2)^{\frac{1}{3}} (x^{-2} y^3)^{\frac{1}{4}}$
 $= x^1 y^{\frac{2}{3}} x^{-\frac{1}{2}} y^{\frac{3}{4}}$
 $= x^{\frac{1}{2}} y^{\frac{8}{12} + \frac{9}{12}}$
 $= x^{\frac{1}{2}} y^{\frac{17}{12}}$

g) $(\sqrt[5]{2a^3 b^4 c^{-2}})^4$
 $= [(2a^3 b^4 c^{-2})^{\frac{1}{5}}]^4$
 $= (2a^3 b^4 c^{-2})^{\frac{4}{5}}$
 $= 2^{\frac{4}{5}} a^{\frac{12}{5}} b^{\frac{16}{5}} c^{-\frac{8}{5}}$
 $= \frac{2^{\frac{4}{5}} a^{\frac{12}{5}} b^{\frac{16}{5}}}{c^{\frac{8}{5}}}$

h) $\sqrt[5]{m^3} \cdot \sqrt[4]{m^5}$
 $= (m^3)^{\frac{1}{5}} (m^5)^{\frac{1}{4}}$
 $= m^{\frac{3}{5}} m^{\frac{5}{4}}$
 $= m^{\frac{3}{5} + \frac{5}{4}}$
 $= m^{\frac{17}{20}}$

i) $\sqrt{\sqrt[3]{8x^5}}$
 $= ((8x^5)^{\frac{1}{3}})^{\frac{1}{2}}$
 $= (8x^{\frac{5}{3}})^{\frac{1}{6}}$
 $= 8^{\frac{1}{6}} x^{\frac{5}{6}}$

j) $(\sqrt[4]{4m^3 n^5})^2$
 $= (4m^3 n^5)^{\frac{2}{4}}$
 $= (4m^3 n^5)^{\frac{1}{2}}$
 $= 4^{\frac{1}{2}} m^{\frac{3}{2}} n^{\frac{5}{2}}$
 $= 2 m^{\frac{3}{2}} n^{\frac{5}{2}}$



Homework

Handout --> Questions that have a square around them.