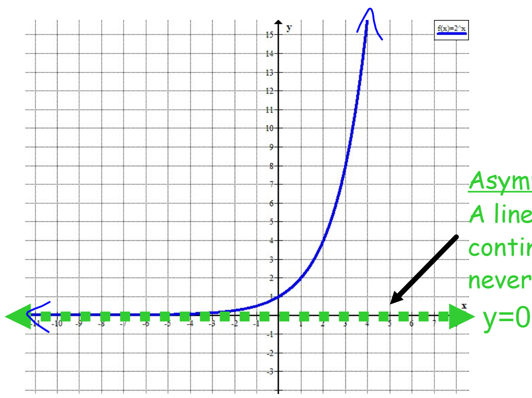
3.1 Exponential Growth and Decay

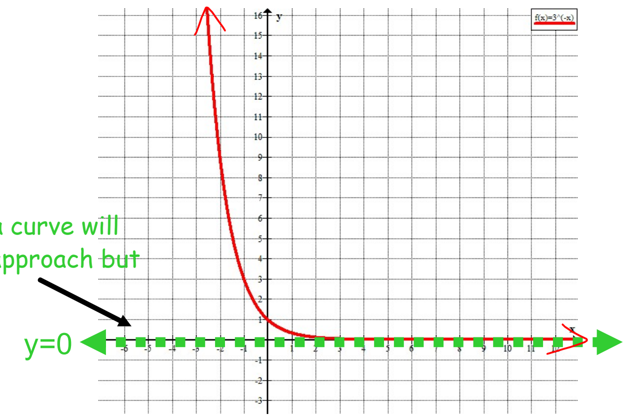
Exponential growth



Notice: The graph is increases slowly then quickly (slope becomes steeper)

Asymptote:
A line that a curve will continually approach but never touch

Exponential Decay



Notice: The graph is decreasing quickly then slowly (slope becomes less steep)

The graph of an exponential growth or decay is a smooth curve that is almost horizontal at one end (approaches an asymptote) and rapidly increases or decreases at the other end.

The equation of an exponential relation contains a constant base and a variable exponent. ex.

$$A = 250(1.04)^{3x}$$

$$y = -3(5)^{x}$$

$$T = t\,(0.76)^{\frac{3}{4}w}$$

Exponential growth or decay can be modelled by an exponential equation:

# of growth/decay periods

Final Amount (Amt. after "x" growth/decay periods)

Note: $x = \frac{t}{d}$ or $x = \frac{t}{h}$

Where d=time it takes to double

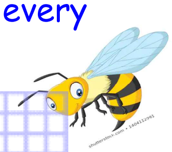
h=time it takes to divide in half

$$A = a_o (b)^x$$

Amount at beginning.

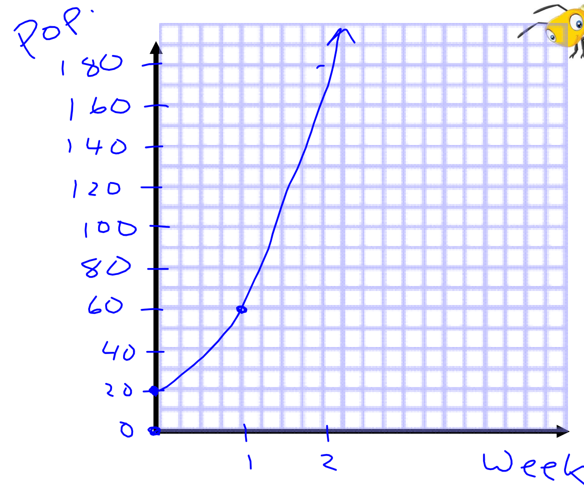
growth factor (b > 1)
decay factor (0< b < 1)

Ex 1 A wasp population starts at an initial population of 20 and triples every week.



a) Complete the table and graph.

Week	Population	First Diff	Second Diff
0	20 ×3	40	80
1	60 ×3	120	240
2	180 ×3	360	720
3	540 ×3	1080	
4	1620		



b) Look for a pattern in the population and differences. What do you notice?

There is NO CONSTANT DIFFERENCE!

$\boxed{\text{It's a COMMON RATIO}}$

c) Find an equation to model this growth. HINT: Use the constant multiplier/ common factor.

$$A = a_0 b^x$$
$$= 20(3)^x$$

Let x be # weeks

d) Use the equation to find the number of wasps after one year.

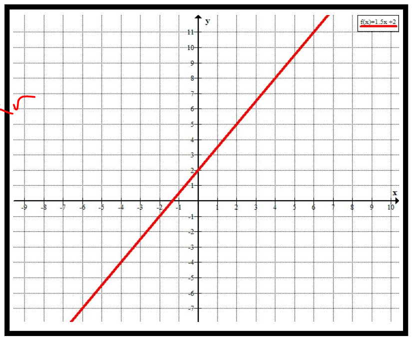
52 weeks in a year

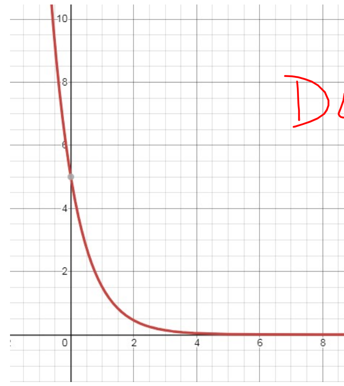
$$A = 20(3)^{52}$$
$$\doteq 1.3 \times 10^{26}$$

∴ That's a LOT of wasps!

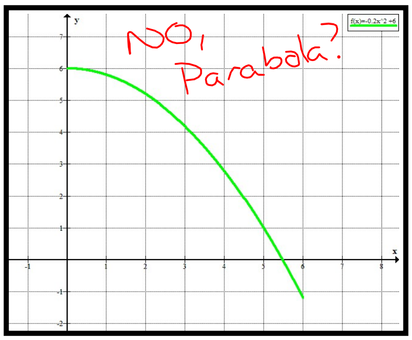
Ex. 2 Which could represent exponential growth or decay?

NO!
Linear

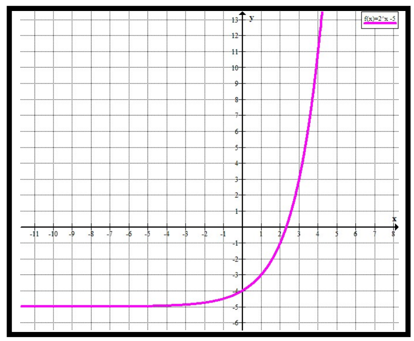




DECAY



NO, Parabola?



GROWTH

$y = 5x^2 + 7x - 3$ NO! QUAD

$A = 400(0.76)^{t/4}$ DECAY

$P = 200(1.07)^{t}$ GROWTH

$P = 7w + 5$ NO -> LINEAR

x	y
0	1
1	5
2	25
3	125
4	625
5	3125

×5 ×5 ×5 ×5 ×5

∴ Exp. Growth

t	A
0	80
1	72
2	64.8
3	58.32
4	52.49
5	47.24

$\frac{72}{80} = 0.9$

$\frac{64.8}{72} = 0.9$

×0.9 ×0.9 ×0.9

∴ Exp DECAY

t	P
0	15
1	16
2	19
3	24
4	29
5	40

$\frac{16}{15} = 1.01$

$\frac{19}{16} = 1.1$

$\frac{24}{19} = 1.2$

∴ NO! Common ratios are different

Ex. 3 The table below shows the amount of radioactive material remaining from a 300 g sample.

Time (hours)	Amount (g)
0	300
1	285
2	270.75
3	257.21
4	244.35
5	232.13

1st diff	ratio of "y" values
-15	0.95 $\frac{285}{300}$
-14.25	0.95
-13.54	0.95
-12.86	0.95
-12.22	

a) Write an exponential equation to model the situation.

$A = a_0 b^x$

$A = 300(0.95)^h$

Let h be # hours

b) Determine an approximate growth/decay rate.

"Factor" $\rightarrow$ 0.95

"Rate" $\rightarrow$ 5% $(1 - 0.95) \times 100\%$

c) Use this equation to determine the amount that will remain after 12 hours.

$A = 300(0.95)^h$

$= 300(0.95)^{12}$

$\doteq 162.1$

$\therefore$ There will be approx. 162g remaining

graph

Ex. 4 Model each situation with an exponential equation.
Define "x" for each.

a) An initial population of 200 tent caterpillars grows by 15% each day.

Growth?
100% + 15%
= 1.15

$A = a_0 b^x$
$A = 200(1.15)^x$

Let x be # of days

b) A car worth $25 000, depreciates in value by 13% each year.

Decay
100% − 13%
= 0.87

$A = a_0 b^x$
$A = 25000(0.87)^x$

Let x be # of years

c) 400 mg of radioactive material deteriorates by 5% every 4 hours.

Decay
100% − 5%
= 0.95

$A = a_0 b^x$
$A = 400(0.95)^{\frac{x}{4}}$

Let x be # of hours

d) A rabbit population of 50 doubles every 6 weeks.

$A = a_0 b^x$
$A = 50(2)^{\frac{x}{6}}$

Let x be # of weeks

Hmk.

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