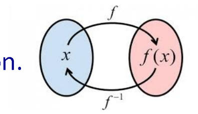
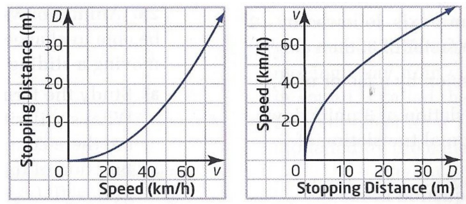
Lesson 2.7: Inverse of a Function

The INVERSE of a relation is the reverse of the original relation.



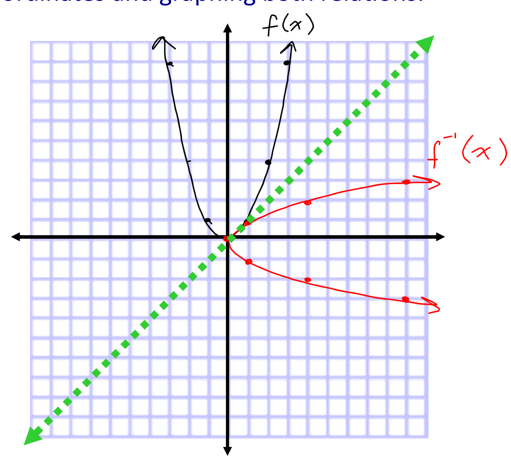


If a function is called *f(x)*, then its inverse function is called $f^{-1}(x)$.

Note: $f^{-1}(x)$ *This is not an exponent!* *Pronounced "the inverse of f at x"* $f^{-1}(x) \neq \frac{1}{f(x)}$

Ex. 1: Explore the inverse of the function f(x) = x^2 by:
a) interchanging the x and y-coordinates and graphing both relations.

Points on Base Function	Points on Inverse
(-3, 9)	(9,-3)
(-2,4)	(4,-2)
(-1,1)	(1,-1)
(0,0)	(0,0)
(1,1)	(1,1)
(2,4)	(4,2)
(3,9)	(9,3)



b) State the domain and range of f(x) and $f^{-1}(x)$. What do you notice?

f(x): $D = \{x \in \mathbb{R}\}$, $R = \{y \in \mathbb{R} \mid y \geq 0\}$

$f^{-1}(x)$: $D = \{x \in \mathbb{R} \mid x \geq 0\}$, $R = \{y \in \mathbb{R}\}$

c) Is $f^{-1}(x)$ a function?

No! It fails the vertical line test.

d) Add the line y = x to your graph. What do you notice?

Reflection across y=x

e) Determine the equation of the inverse function by interchanging x and y and solving for y.

$f(x) = x^2$

$y = x^2$

Interchange x & y

$x = y^2$

$\pm\sqrt{x} = y$

$\therefore f^{-1}(x) = \pm\sqrt{x}$

The Big Ideas

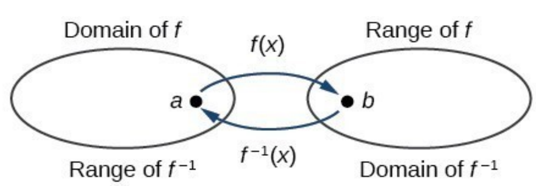
There are three ways to find the inverse of a function:

1) Given the coordinates, interchange x and y. $(a,b) \Rightarrow (b,a)$ or $f(a) = b \Rightarrow f^{-1}(b) = f(a)$

2) Given the equation, interchange x and y and solve for y.

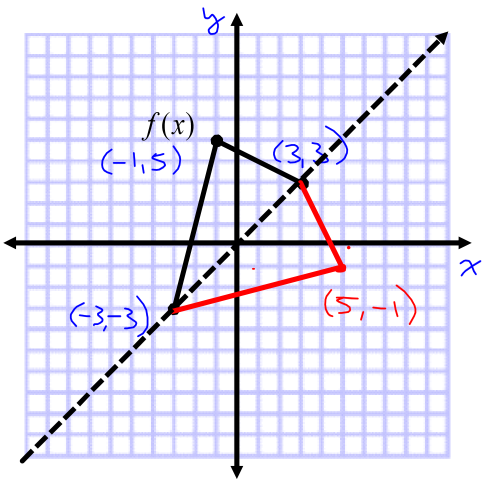
3) Given the graph, reflect in the line y = x.

NOTE: Not all inverse relations are functions.
The domain of a function is the range of its inverse.
The range of a function is the domain of its inverse.

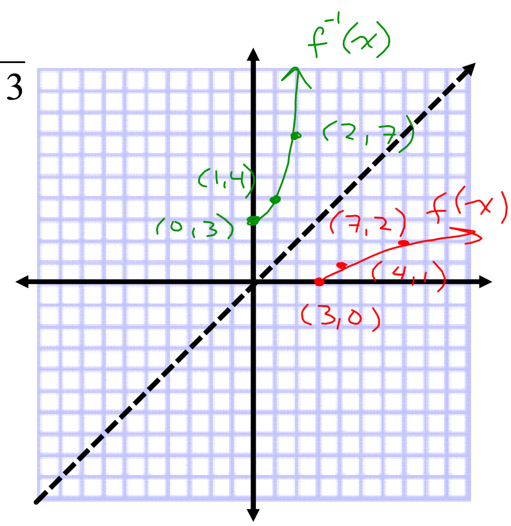


Ex. 2: Given f(x), graph $f^{-1}(x)$.

a)



b) $f(x)=\sqrt{x-3}$



What is the mapping notation for the inverse of a function?

$(x,y)\rightarrow(y,x)$

Which points are invariant?

All points on $y=x$

What do you notice about the domain of f(x) and the range of $f^{-1}(x)$?

$D=\{x\in\mathbb{R}\mid x\geq 3\}$

$R=\{y\in\mathbb{R}\mid y\geq 3\}$

Ex. 3: Determine the equation of the inverse for each of the following.

Process
1. Write f(x) as y.
2. Interchange x and y.
3. Solve for y.
4. Rewrite as $f^{-1}(x)$.

a) $f(x) = 5x - 2$

① $y = 5x - 2$

② $x = 5y - 2$

③ $x + 2 = 5y$

$\frac{x+2}{5} = y$

④ $f^{-1}(x) = \frac{x+2}{5}$

Sometimes it takes some extra work to go between f(x) and $f^{-1}(x)$!

b) $f(x) = \dfrac{1}{x+5}$

$y = \frac{1}{x+5}$

$x = \frac{1}{y+5}$

$(y+5)x = 1$

$y + 5 = \frac{1}{x}$

$\rightarrow y = \frac{1}{x} - 5$

$\therefore f^{-1}(x) = \frac{1}{x} - 5$

c) $f(x) = \sqrt{x+4}$

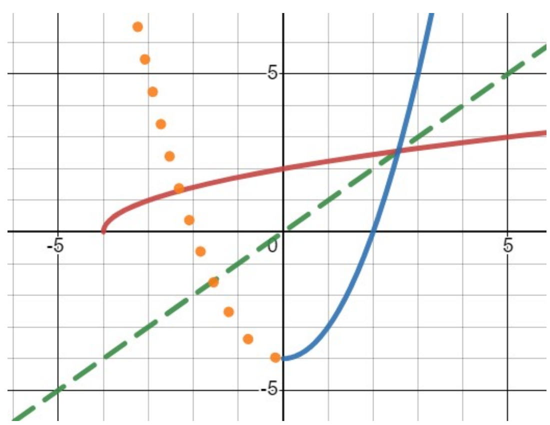
$y = \sqrt{x+4}$

$x = \sqrt{y+4}$

$x^2 = y + 4$

$x^2 - 4 = y$

$f^{-1}(x) = x^2 - 4$ $D = \{x \in \mathbb{R} \mid x \geq 0\}$



Notice that the domain of $f^{-1}(x)$ needs to be restricted so that it is equivalent to the range of f(x).

Ex. 4: Given $f(x) = -3(x-4)^2 + 2$:

$V(4,2)$

a) Determine $f^{-1}(x)$.

$y = -3(x-4)^2 + 2$

$x = -3(y-4)^2 + 2$

$x - 2 = -3(y-4)^2$

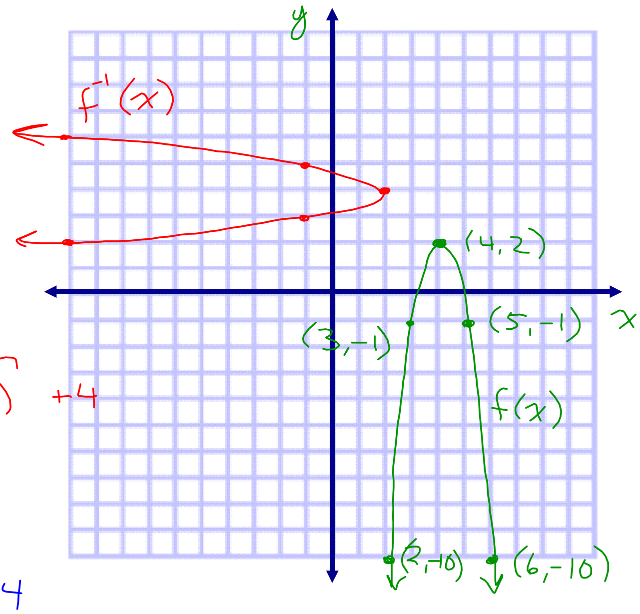
$\frac{x-2}{-3} = (y-4)^2$

$\pm\sqrt{\frac{x-2}{-3}} = y - 4$

$\pm\sqrt{\frac{x-2}{-3}} + 4 = y$

$\pm\sqrt{-\frac{1}{3}(x-2)} + 4$

$\therefore\ f^{-1}(x) = \pm\sqrt{\frac{x-2}{-3}} + 4$



b) Graph $f(x)$ and $f^{-1}(x)$.

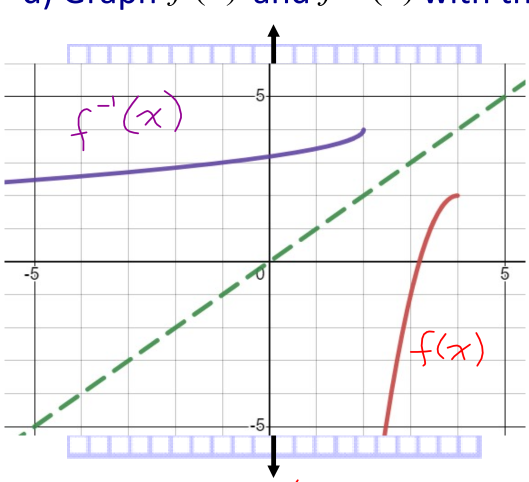
c) Restrict the domain of $f(x)$ to one branch so that $f^{-1}(x)$ is also a function.

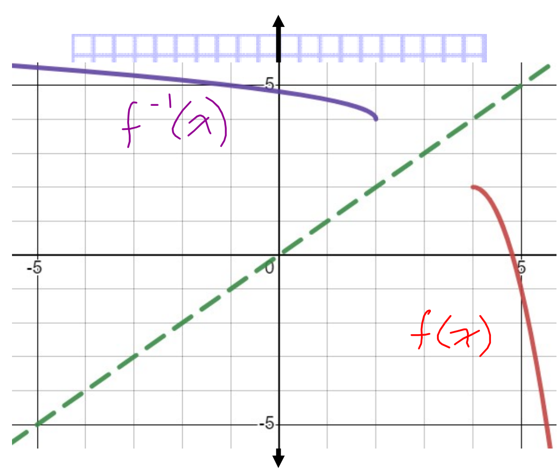
🤔 When restricting the domain identify the x-coord of the vertex.

For the right branch

$D = \{x \in \mathbb{R} \mid x \geq 4\}$

d) Graph $f(x)$ and $f^{-1}(x)$ with the restricted domains.





LEFT BRANCH

$D = \{x \in \mathbb{R} \mid x \leq 4\}$

RIGHT BRANCH

$D = \{x \in \mathbb{R} \mid x \geq 4\}$

Homework

p. 137 #C3, 1a, 2a, 3bc, 4d, 5d, 7c (no technology), 11, 15bc, 18, 20, 21