Lesson 2.6A: Combinations of Transformations

Summary of Transformations

If y = f(x), then describe the effect of a, k, d and c.

d represents *shift left/right* horizontal translation

if d < 0 left "d" units

if d > 0 right "d" units

c represents *Shift up/down* vertical translation

if c < 0 down "c" units

if c > 0 up "c" units

$$y = a\, f[\, k\,(x - d)\,] + c$$

a represents vertical stretch/compression and/or reflection

ex: $a = \frac{1}{2}$ is compression by 2

if 0 < a < 1 vertical compression by " $\frac{1}{a}$ "

if a > 1 vertical stretch by "a"

if a < 0 reflection in the x-axis

k represents horizontal stretch/compression and/or reflection

if 0 < k < 1 horiz. stretch of " $\frac{1}{k}$ "

if k > 1 horiz. compression of "k"

if k < 0 reflection in the y-axis

Order of Transformations

1ˢᵗ: Transformations that are multiplied/divided
- Stretching /Compressing
- Reflecting

2ⁿᵈ: Transformations that are added/subtracted.
- Translations

Stretch First!

Ex. 1: Describe the following transformations on a function $y = f(x)$.

Function	Transformations to apply first	Transformations to apply second
$y = -2 f(x - 3) + 1$	• Reflection in the x-axis • Vertical stretch of 2	• Horizontal translation to the right 3 units • Vertical translation up 1 unit
$y = f(3(x + 2)) - 5$	• Horizontal compression by 3	• Horizontal translation to the left 2 units • Vertical translation down 5 units
$y = -5 f(-2(x - 4)) - 1$	• Reflection in the x-axis • Vertical stretch of 5 • Reflection in the y-axis • Horizontal compression by 2	• Horizontal translation to the right 4 units • Vertical translation down 1 unit

If the transformed equation is not given in <u>standard form</u>, you will have to <u>factor</u> in order to determine the correct order of the transformations.

$$y = a \, f[\, k \, (x - d) \,] + c$$

Ex. 2: Identify the transformations that have been applied to each of the following base functions.

a) $y = f(3x + 6)$

Always FACTOR!

$y = f\left[3(x+2)\right]$
         ①   ②

① Horizontal compression by 3

② Shift left 2

b) $y = \sqrt{-x + 5}$

$y = \sqrt{-(x-5)}$
         ①   ②

① Horz. reflection in y-axis

② Shift right 5

$$y = a\,f[\,k\,(x - d)\,] + c$$

$$(x, y) \rightarrow \left(\frac{x}{k} + d,\, ay + c\right)$$

Ex. 3: Given the graph of f(x), transform the function as indicated.

g(x) = - f(2x + 6) - 2

$$= -f\left[2(x+3)\right] - 2$$
②   ①   ③   ④

$$(x, y) \rightarrow \left(\frac{x}{2} - 3,\, -y - 2\right)$$

① Horz. Compression by 2
② Reflection in x-axis
③ Shift left 3
④ Shift down 2
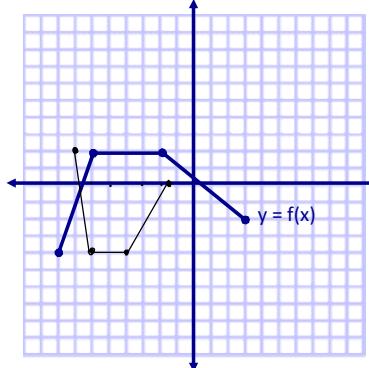
$$(-2, 2) \rightarrow (-4, -4)$$
$$(-8, -4) \rightarrow (-7, 2)$$
$$(-6, 2) \rightarrow (-6, -4)$$
$$(3, -2) \rightarrow \left(-\frac{3}{2}, 0\right)$$

y = f(x)

Ex. 4: Write the equation of the transformed function using the given base function and transformations.

a)   f(x) = x² , reflected in the x-axis, vertical stretch by 3, left 6, down 2.
① $-x^2$ ②   ③

① $f(x) = -x^2$

② $f(x) = -3x^2$

③ $f(x) = -3(x+6)^2 - 2$

b) $f(x) = \dfrac{1}{x}$, horizontal stretch by 2, reflection in the y-axis, right 7 , up 3.
①   ②   ③

① $f(x) = \dfrac{1}{\frac{1}{2}x}$

② $f(x) = \dfrac{1}{-\frac{1}{2}x}$

③ $f(x) = \dfrac{1}{-\frac{1}{2}(x-7)} + 3$

Simplifies to

$$f(x) = \dfrac{-2}{x-7} + 3$$

c) $f(x) = \sqrt{x}$, horizontal compression by 3, reflection in the x-axis,
③ reflection in the y-axis, left 6, down 2.
①   ②   ④

① $f(x) = \sqrt{3x}$

② $f(x) = -\sqrt{3x}$

③ $f(x) = -\sqrt{-3x}$

④ $f(x) = -\sqrt{-3(x+6)} - 2$

Ex. 5: Given f(x), write the corresponding equation and graph the transformed function.

a) $f(x) = x^3$, graph $g(x) = -2f(3 - x) + 4$

$$= -2f(-x+3) + 4$$
$$= -2f[-(x-3)] + 4$$

$$(x, y) \rightarrow (-x+3, -2y+4)$$

$(0,0) \rightarrow (3,4)$
$(1,1) \rightarrow (2,2)$
$(2,8) \rightarrow (1,-12)$
$(-1,-1) \rightarrow (4,6)$
$(-2,-8) \rightarrow (5,20)$
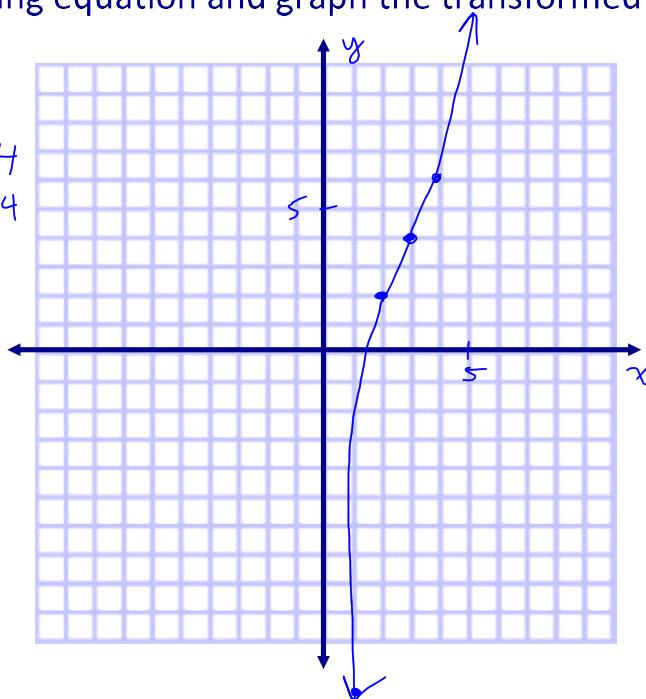
b) $f(x) = \sqrt{x}$, graph $g(x) = f(-2x + 6) - 5$

$$= f[-2(x-3)] - 5$$

$$(x, y) \rightarrow (-\tfrac{x}{2}+3, y-5)$$

$(0,0) \rightarrow (3,-5)$
$(1,1) \rightarrow (\tfrac{5}{2}, -4)$
$(4,2) \rightarrow (1,-3)$
$(9,3) \rightarrow (-\tfrac{3}{2}, -2)$
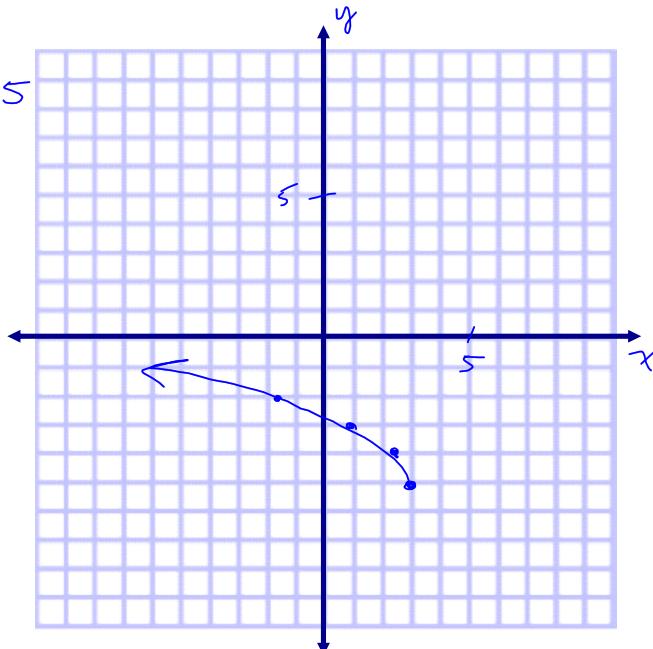
c) $f(x) = \dfrac{1}{x}$, graph $g(x) = \dfrac{1}{2}f(-x + 3) - 4$

$= \dfrac{1}{2}f\left[-(x-3)\right] - 4$

$(x, y) \rightarrow \left(-x+3, \dfrac{1}{2}y - 4\right)$

$\left(\dfrac{1}{2}, 2\right)$

$(1, 1)$

$\left(2, \dfrac{1}{2}\right)$

$\left(-\dfrac{1}{2}, -2\right)$

$(-1, -1)$

$\left(-2, -\dfrac{1}{2}\right)$
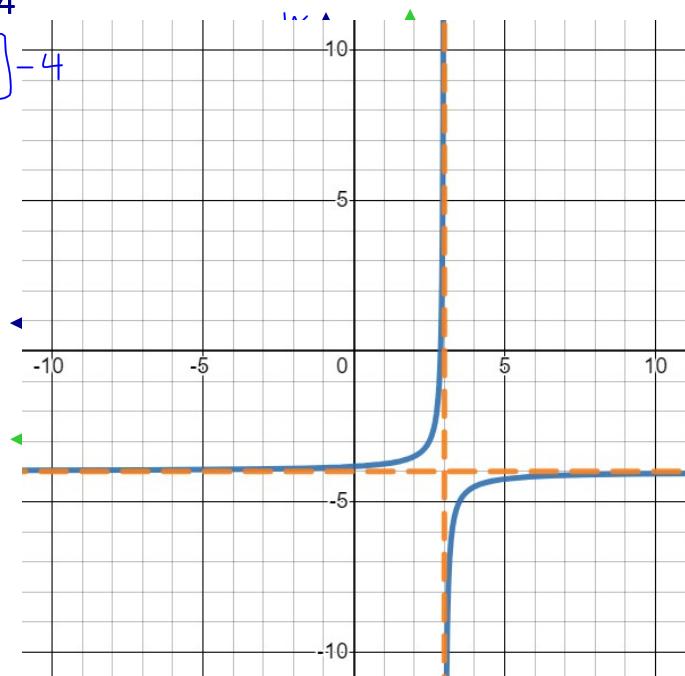
Homework
p.129 #C2, 1ace, 2cde, 3abc, 4bcd, 5cdf, 7ac, 11, 13b

Extra Practice 2.6A