Lesson 2.5: Stretches/Compressions of Functions

Gizmo

Part A: Vertical Stretches & Compressions $g(x) = af(x)$

$g(x) = af(x)$ is the graph of $f(x)$ that has been vertically stretched by a factor of "a".

If $a > 1$, then the graph is vertically stretch.

If $0 < a < 1$, then the graph is vertically compression.

Ex. 1: Given $f(x)$ as shown, graph:

a) $g(x) = 3f(x)$ Vertical stretch by 3

(x,y) -> (x, 3y)

(-3,0) → (-3, 0)

(-2,1) → (-2, 3)

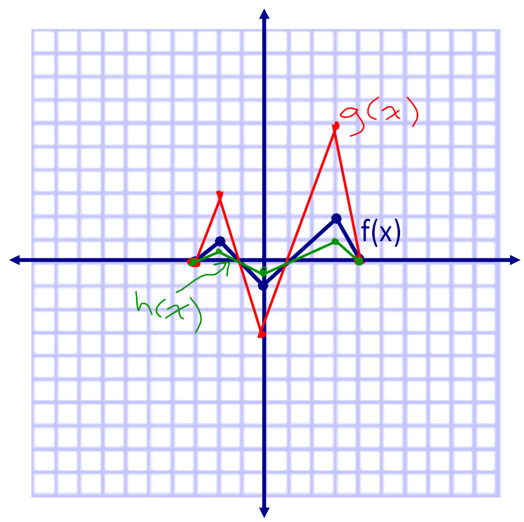
(0,-1) → (0, -3)

(3,2) → (3, 6)

(4,0) → (4, 0)

b) $h(x) = \frac{1}{2}f(x)$ Compression by 2

(x,y) -> (x, $\frac{1}{2}$y)



Which points are invariant? ⇒ Points lying on the x-axis (y-coord = 0)

Ex. 2: Given $f(x) = x^2$ write equations to represent g(x) and h(x) and graph :

a) $g(x) = 2f(x)$ Vert. stretch by 2

(x,y) -> (x, 2y)

(1,1) → (1, 2)

(2,4) → (2, 8)

(3,9) → (3, 18)

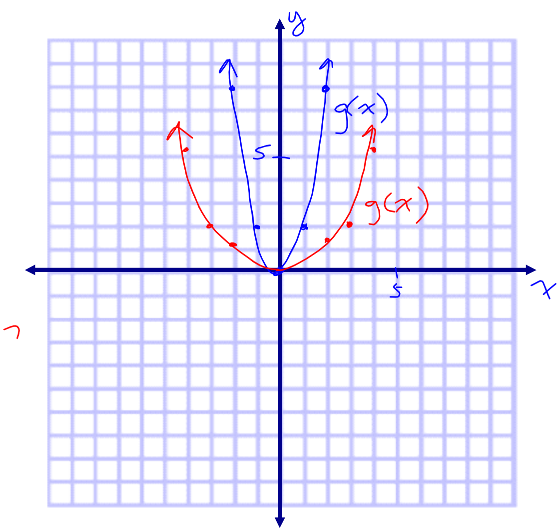
b) $g(x) = \frac{1}{3}f(x)$ Vert. Compression by 3

(x,y) -> (x, $\frac{y}{3}$)

(2,4) → (2, $\frac{4}{3}$)

(3,9) → (3, 3)

(4,16) → (4, $\frac{16}{3}$)



What do you notice about the domain and range? ⇒ The domain is not affected by a vertical transformation. The range is affected.

Part B: Horizontal Stretches & Compressions $g(x) = f(kx)$

$g(x) = f(kx)$ is the graph of $f(x)$ that has been horizontally stretched by a factor of " $\frac{1}{k}$ ".

If $k > 1$, then the graph is horizontally compression.

If $0 < k < 1$, then the graph is horizontally stretch.

Note: k does the opposite of what you naturally think since it is inside the function.
Note: Textbook uses incorrect terminology for both vertical and horizontal compressions!

Ex. 3: Given $f(x)$, graph:

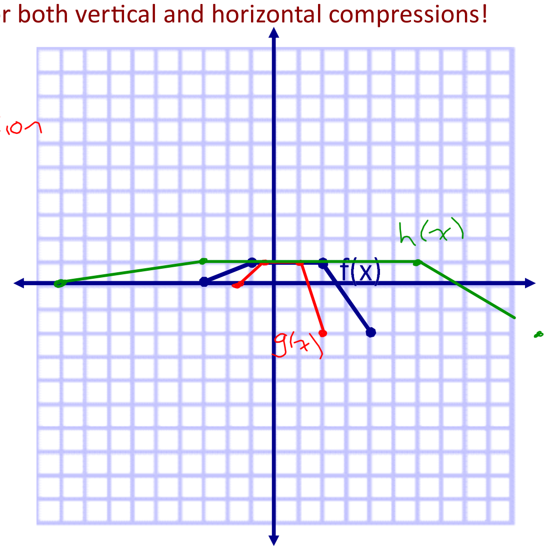
a) $g(x) = f(2x)$ Horz. compression by 2

(x,y) -> ($\frac{x}{2}$, y)

(-3, 0) -> ($-\frac{3}{2}$, 0)

b) $h(x) = f\left(\frac{1}{3}x\right)$ Horz. stretch by 3

(x,y) -> (3x, y)



Which points are invariant? ⇒ Points lying on the y-axis (x-coord = 0)

Ex. 4: Given $f(x) = \sqrt{x}$ write equations to represent g(x) and h(x) and graph :

a) $g(x) = f(4x)$ Horz. compression by 4

(x,y) -> ($\frac{x}{4}$, y)

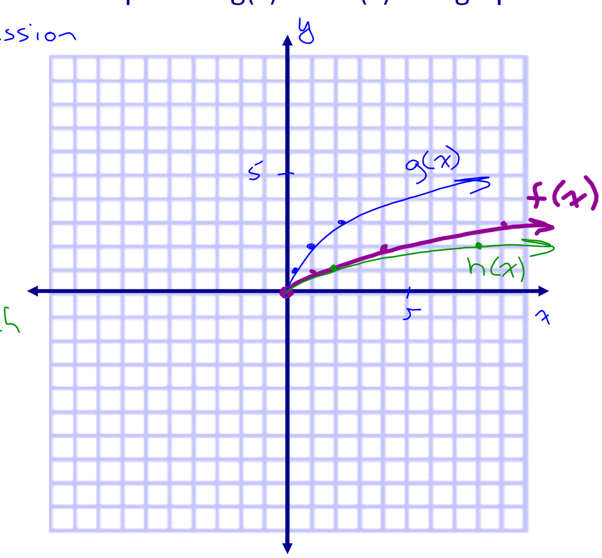
(1, 1) -> ($\frac{1}{4}$, 1)

(4, 2) -> (1, 2)

(9, 3) -> ($\frac{9}{4}$, 3)

b) $h(x) = f\left(\frac{1}{2}x\right)$ Horz. stretch by 2

(x,y) -> (2x, y)



What do you notice about the domain and range? ⇒ The range is not affected by a horizontal transformation. The domain is affected.

Part C: Combining Horizontal & Vertical Stretches & Compressions

Ex. 5: Given $f(x) = |x|$:

a) Write an equation to represent $g(x) = 2f(2x)$.

① ②

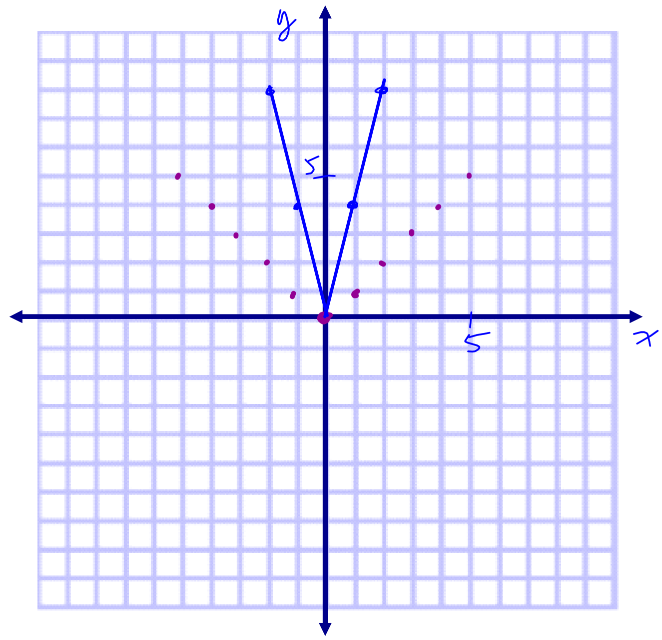
b) Describe the transformations.

① Vertical stretch by 2

② Horizontal compression by 2

c) Graph $g(x) = 2f(2x)$.

$(x, y) \rightarrow \left(\frac{x}{2}, 2y\right)$



d) State the domain and range.

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \geq 0\}$

Ex. 6: Given that f(x) = (2x)^2 is a parabola that has been horizontally compressed by a factor of 2 , can you describe a different transformation that would give the SAME graph?

$f(x) = (2x)^2$

$= 4x^2$

Vertical stretch of 4

Equivalent functions have the same graph

Homework
p.119 #C1, C3, 2bcd/i/iii/iv,
3, 4, 6, 7abcde, 13 (use desmos)

Extra Practice 2.5