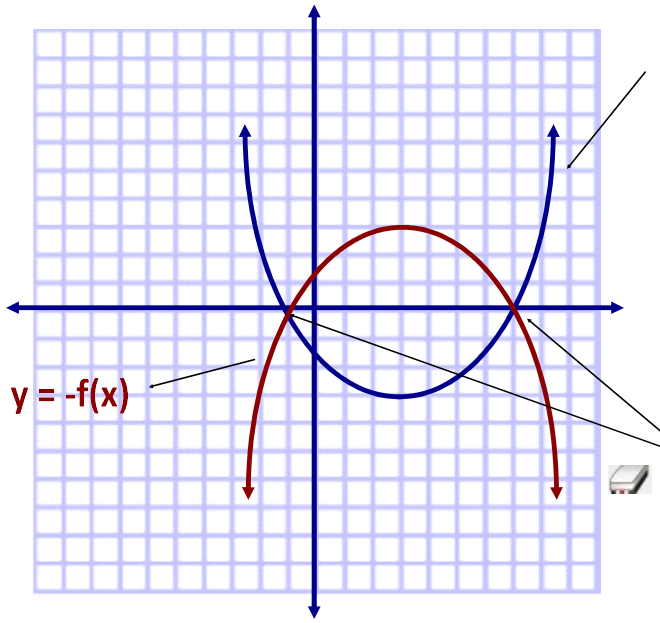
Lesson 2.4: Reflections

A reflection creates a mirror image.

A. Reflection in the x-axis (vertical reflection).



What do you notice about the y-values?

Flip sign

What do you notice about the x-values?

Do not change

Invariant points are on the x-axis.

Ex. 1:

a) Graph $f(x) = (x - 3)^2 + 5$

Vertex (3, 5)

$a = 1$

Pattern

$\rightarrow 1 \uparrow 1$

$\rightarrow 2 \uparrow 4$

$\rightarrow 3 \uparrow 9$

b) Find the equation of $g(x) = -f(x)$.

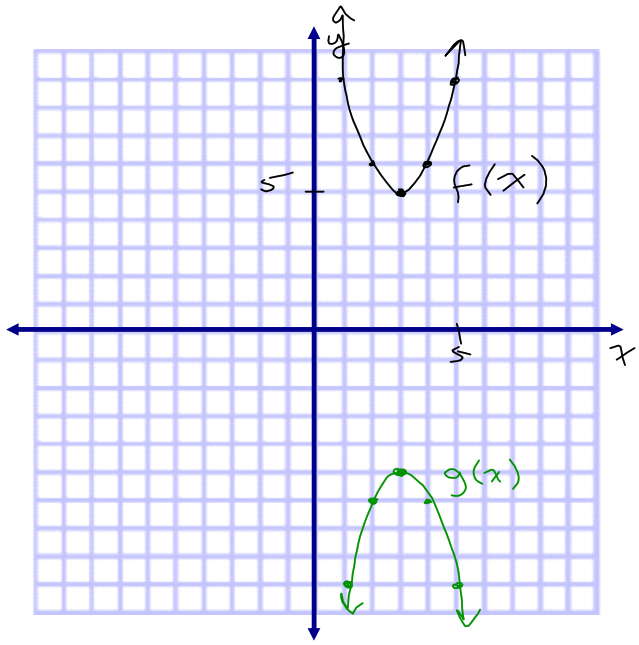
$g(x) = -[(x-3)^2 + 5]$

$= -(x-3)^2 - 5$

c) Graph g(x).

$a = -1$

V (3, -5)



d) List any invariant points.

None

e) State the domain and range of f(x) and g(x).

f(x): D= $\{x \in \mathbb{R}\}$

R= $\{y \in \mathbb{R} \mid y \geq 5\}$

g(x): D= $\{x \in \mathbb{R}\}$

R= $\{y \in \mathbb{R} \mid y \leq -5\}$

Ex. 2:

(0,0)
(1,1)
(4,2)
(9,3)

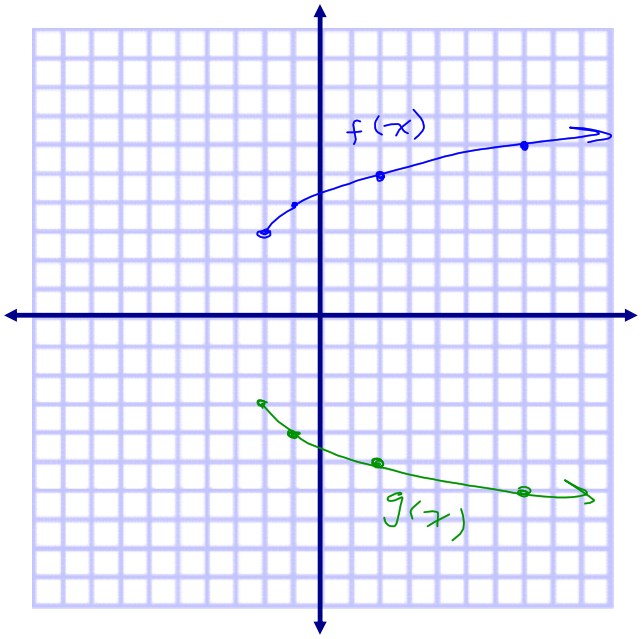
a) Graph $f(x) = \sqrt{x+2}+3$

Parent $\sqrt{x}$ LEFT 2 UP 3

b) Find the equation of g(x) = -f(x).

$g(x) = -[\sqrt{x+2}+3]$

$= -\sqrt{x+2} - 3$



c) Graph g(x).

Reflection in x-axis

d) List any invariant points.

None

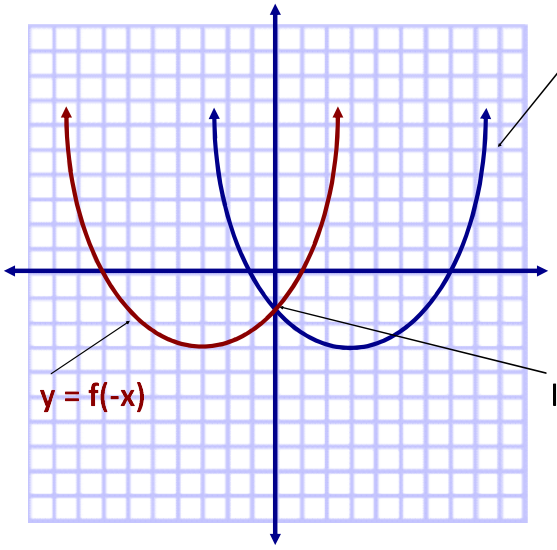
e) State the domain and range of f(x) and g(x).

f(x): $D=\{x \in \mathbb{R} \mid x \geq -2\}$ $R=\{y \in \mathbb{R} \mid y \geq 3\}$

Same

g(x): $D=\{x \in \mathbb{R} \mid x \geq -2\}$ $R=\{y \in \mathbb{R} \mid y \leq -3\}$

B. Reflection in the y-axis (horizontal reflection).



What do you notice about the x-values?

Change (flipped)

What do you notice about the y-values?

Do not change

Invariant points are on the y-axis.

Ex. 3:

a) Graph $f(x) = (x + 4)^2$

Vertex (-4, 0)

Replace "x" with "-x".

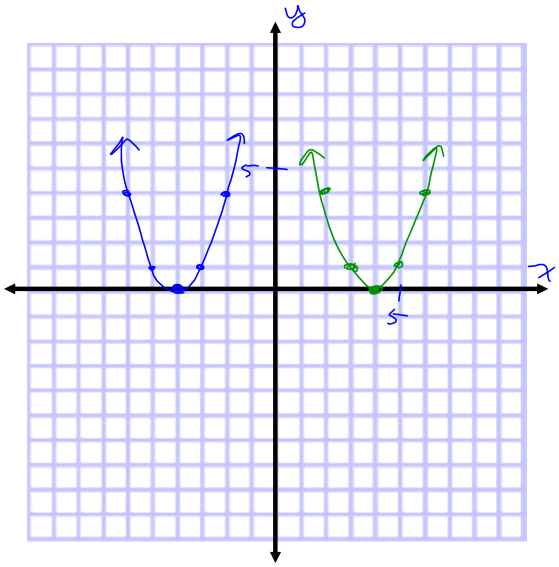
b) Find the equation of g(x) = f(-x).

$g(x) = (-x+4)^2$ [2]

$= [-(x-4)]^2$

$= (-1)^2 (x-4)^2$

$= 1(x-4)^2$



c) Graph g(x).

Vertex (4, 0)

d) List any invariant points.

All invariant points for horz. reflection are y-ints

Set $x = 0$

$f(0) = (0+4)^2$

$= 16$

$\therefore (0, 16)$ is invariant

e) State the domain and range of f(x) and g(x).

f(x): $D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y \geq 0\}$

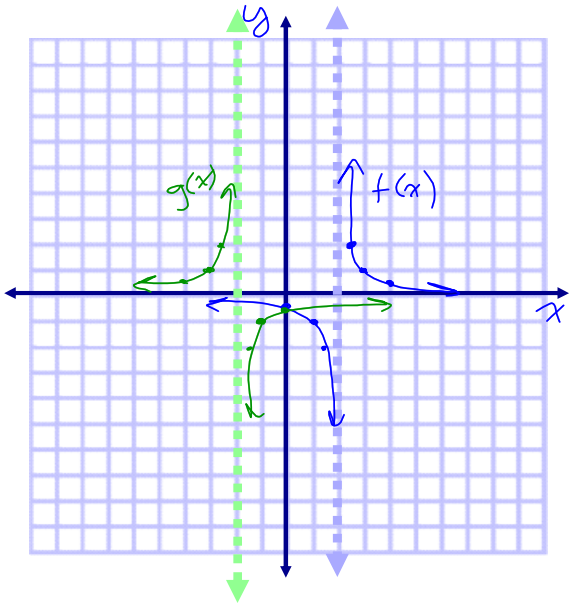
g(x): $D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y \geq 0\}$

Ex. 4

a) Graph $f(x) = \dfrac{1}{x-2}$

Base $\frac{1}{x}$ → 2 RIGHT



b) Find the equation of g(x) = f(-x).

$g(x) = \dfrac{1}{-x-2}$

$= \dfrac{1}{-(x+2)}$

Vert. Reflection → $= -\dfrac{1}{x+2}$

c) Graph g(x).

d) List any invariant points.

y-ints!

$f(0) = \dfrac{1}{0-2}$ $(0, -\frac{1}{2})$

$= -\frac{1}{2}$

e) State the domain and range of f(x) and g(x).

f(x): $D = \{x \in \mathbb{R} \mid x \neq 2\}$
$R = \{y \in \mathbb{R} \mid y \neq 0\}$

g(x): $D = \{x \in \mathbb{R} \mid x \neq -2\}$
$R = \{y \in \mathbb{R} \mid y \neq 0\}$

Summary

If y = f(x), then:

y = - f(x) represents a reflection in x-axis.
Under a reflection in the x-axis, the point (x, y) ⇒ (x, -y).
Invariant points are located x-ints.

y = f(-x) represents a reflection in y-axis.
Under a reflection in the y-axis, the point (x, y) ⇒ (-x, y).
Invariant points are located y-ints.

y = - f(-x) represents a reflection in x-axis and y-axis. The order of these reflections does not matter. Under these reflections, the point (x, y) ⇒ (-x, -y).

Ex. 5:

Given f(x) = $|x-3|$

a) Graph f(x).

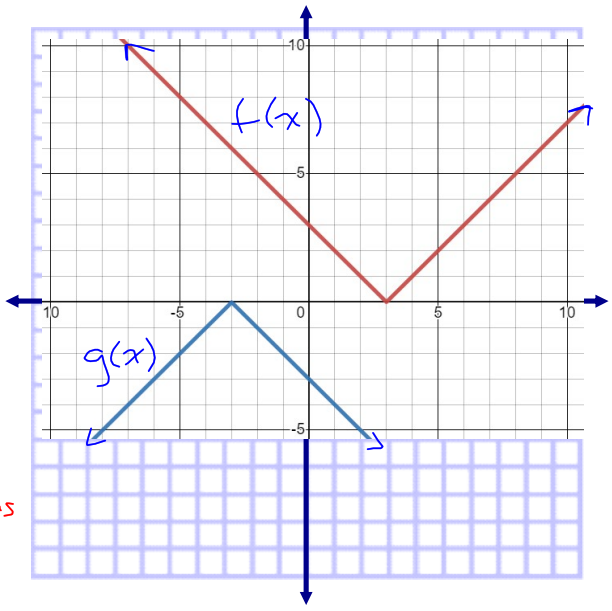
b) Find the equation of g(x) = - f(-x).
Graph g(x).

$g(x) = -|-x-3|$

$= -|-(x+3)|$

ABS Value means positive

$= -|x+3|$



Homework

p. 110 #C1, 1ef, 2cef, 3ef, 4acdf (textbook does not fully simplify equations - you are expected to), 5, 8a (do not use tech - solve algebraically), 12 abcd, 13 (omit -f(-x))

Extra Practice 2.4