2.1 Functions and Equivalent Expressions

In grade 12, you will learn how to graph polynomial functions and rational functions. To prepare for this work, in grade 11 you learn how to simplify rational expressions.

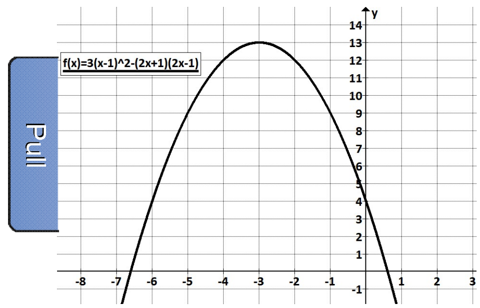
Ex. 1 Simplify the polynomial expression: $3(x-1)^2 - (2x+1)(2x-1)$

$(x-1)(x-1)$

$= 3(x^2-2x+1) - (4x^2-1)$
$= 3x^2-6x+3 - 4x^2+1$
$= -x^2-6x+4$

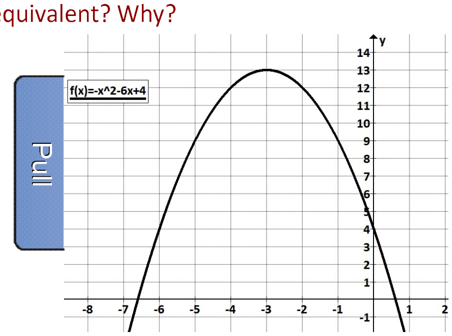
What do you think the graph of the polynomial function $f(x) = 3(x-1)^2 - (2x+1)(2x-1)$ will look like? Why?

Quadratic
Opens Down



Will the graph of the polynomial function $f(x) = -x^2 - 6x + 4$ be equivalent? Why?

Yes!
- Same expression
- Same graph



Note: Talk about domain here... and whether substituting values for x is enough to show equivalency.

ex: $y = -x^2-6x+4$ $\qquad$ $y = 2x+11$

$x = -1$

$y = -(-1)^2 - 6(-1) + 4$
$= -1 + 6 + 4$
$= 9$

$y = 2(-1) + 11$
$= -2 + 11$
$= 9$

Not sufficient to simply show equivalency.
ex: These are NOT the same
Despite same results for $x=-1$

What is a rational expression and how do we simplify them?

- Recall that a rational number is any number that can be expressed in the form $\frac{a}{b}$ where $b \neq 0$.

- Likewise, a rational expression is the quotient of two polynomial expressions $\frac{p(x)}{q(x)}$ where $q(x) \neq 0$.

$\frac{P(x)}{0}$ ← UNDEFINED

- To simplify rational numbers,

we divide out common factors.

$$\frac{\cancel{20}^{\,4}}{\cancel{25}_{\,5}} = \frac{4}{5}$$

The same process is used to simplify rational expressions.

Ex. 2 Simplify the polynomial expression:

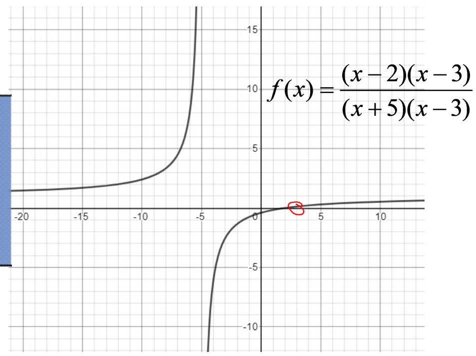
$$\frac{x^2-5x+6}{x^2+2x-15}$$ Factor!

$$=\frac{\cancel{(x-3)}(x-2)}{(x+5)\cancel{(x-3)}}$$

$$=\frac{x-2}{x+5}$$

Let's look at the graph of this rational function:

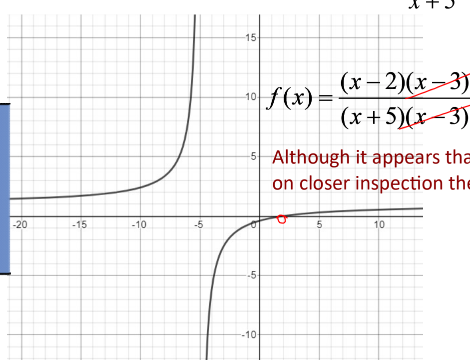
Pull



What is happening at $x=-5$?

Is the graph of the rational function $f(x)=\frac{x-2}{x+5}$ equivalent?

Pull



Although it appears that the functions are equivalent, on closer inspection there is an important difference.

Pull

Ex. 3 Simplify each expression and determine any restrictions.

a) $\dfrac{5x^2+10x}{2x^2+4x}$

$= \dfrac{5x\cancel{(x+2)}}{2x\cancel{(x+2)}}$ Look for restrictions HERE!

$= \dfrac{5}{2}, \; x \neq -2, 0$

Process
1. Factor the numerator and denominator.
2. Divide out any common factors.
3. State restrictions.

To state restrictions, determine the value(s) of the variable that make the denominator equal to zero.
Restrictions are placed after factoring but before simplifying.

b) $\dfrac{2x-1}{4-8x}$

$= \dfrac{2x-1}{4(1-2x)}$

$= \dfrac{\cancel{2x-1}}{-4\cancel{(2x-1)}}$

$= -\dfrac{1}{4}, \; x \neq \dfrac{1}{2}$

c) $\dfrac{8x^3-4x^2+6x}{2x^2}$ Can't factor

$= \dfrac{\cancel{2x}(4x^2-2x+3)}{\cancel{2x}x^2}$

$= \dfrac{4x^2-2x+3}{x}, \; x \neq 0$

d) $\dfrac{x^2+x}{x^2+2x+1}$

$= \dfrac{x\cancel{(x+1)}}{(x+1)\cancel{(x+1)}}$

$= \dfrac{x}{x+1}, \; x \neq -1$

e) $\dfrac{2x^2+7x-15}{4x^2-9}$

$= \dfrac{(x+5)\cancel{(2x-3)}}{(2x+3)\cancel{(2x-3)}}$

$= \dfrac{x+5}{2x+3}, \; x \neq \pm\dfrac{3}{2}$

M -30
A 7
N $\dfrac{3}{10}$ $\dfrac{2}{-3}$
$\dfrac{1}{5}$

$2x+3=0$ $2x-3=0$
$x=-\dfrac{3}{2}$ $x=\dfrac{3}{2}$

Ex. 4 State the restrictions.

a) $\dfrac{4xy^3}{12x^5y^2}$

$x \neq 0$

$y \neq 0$

b) $\dfrac{1}{x^2+9}$

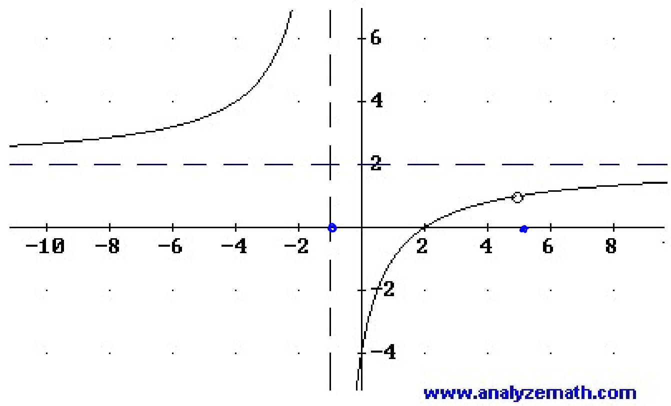
$x^2+9 \overset{?}{=} 0$

$x^2 = -9$

$x = \sqrt{-9}$

$\therefore$ NO RESTRICTIONS

c)



$x = -1, 5$

Ex. 5 Write a rational expression is one variable such that the restrictions are $x \neq \dfrac{-1}{3}, \dfrac{1}{2}$.

$x = -\frac{1}{3}$

$3x = -1$

$3x+1 = 0$

$x = \frac{1}{2}$

$2x = 1$

$2x-1 = 0$

$\therefore \dfrac{1}{(3x+1)(2x-1)}$

HOMEWORK
Pg. 83 #C2, C3 1d, 2d, 3, 4, 6, 13
+ Additional HW Handout Lesson 2.1