1.2B 3U Exponents Practice

3.2 Exercise

1 Evaluate.

(a) -3^{-2} (b) $(-3)^{-2}$ (c) -3^0 (d) $(-3)^0$

(e) $2^2 \times 3^{-1}$ (f) $(2^2 \times 3)^{-1}$ (g) $(-3)^2$ (h) $-(-3)^{-2}$

2 Evaluate.

(a) 2^{-3} (b) $3^0 \times 2^{-2}$ (c) 5^{-2} (d) -3×3^{-1} (e) $\left(\frac{1}{3}\right)^{-2}$

(f) $(-7)^0$ (g) $3^0 - 3^{-1}$ (h) $-3^{-2} + 3^0$ (i) $8 \div 2^{-2}$ (j) $\left(\frac{3^{-1}}{2^{-1}}\right)^{-2}$

3 Express each of the following with a positive exponent.

(a) x^{-2} (b) y^{-3} (c) a^{-7} (d) b^{-5} (e) $\frac{1}{x^{-2}}$ (f) $\frac{1}{a^{-1}}$

(g) $\frac{1}{c^{-4}}$ (h) $2x^{-4}$ (i) ab^{-5} (j) $\frac{x^{-3}}{y^{-4}}$ (k) $4a^{-2}$ (l) $\frac{3x^{-4}}{2y^{-2}}$

4 Express each of the following with all variables in the numerator.

(a) $\frac{1}{x^3}$ (b) $\frac{x}{y^4}$ (c) $\frac{3a}{b^3}$ (d) $\frac{4}{a^{-3}}$ (e) $\frac{3}{a^{-2}b^{-1}}$ (f) $\frac{3}{(x+y)^{-2}}$

5 Simplify each of the following.

(a) $a^5 \times a^{-3}$ (b) $(m^{-3})^2$ (c) $a^5 \div a^{-8}$ (d) $(3a)^{-1}$

(e) $x^5 \div x^{-1}$ (f) $(xy^2)^{-1}$ (g) $(a^{-1})^3 \div a^{-1}$ (h) $\left(\frac{b^3}{b^{-3}}\right)^{-2}$

6 Evaluate.

(a) $(3^0 + 3^2)^{-2}$ (b) $2^4 + \left(\frac{1}{2}\right)^{-3}$ (c) $4^{-1}(4^2 + 4^0)$

(d) $(3^3 - 3^2)^2 \div 3^2$ (e) $(5^7 \div 5^{-2})(5^0 - 1)$ (f) $\frac{2^5}{3^{-2}} \times \frac{3^{-1}}{2^4}$

7 Evaluate.

(a) $(-3)^{-2}(3^3 - 3^4)$ (b) $\frac{3^{-2} \times 2^{-3}}{3^{-2} + 2^{-3}}$ (c) $\frac{4^{-1} + 3^{-2}}{3^{-2} + 2^{-3}}$ (d) $\frac{5^{-1} - 2^{-2}}{5^{-1} + 2^{-2}}$

8 Simplify. Write each as a power with a single base.

(a) $(8^{2-2n})(16^{3-n}) \div (4^{2n})^{-1}$ (b) $[3^{-2}(27^m \div 9^{-m})]^{-1}$

9 If $x = -1$ and $y = 2$, find the value of each of the following.

(a) $\frac{16x^{-3}y^{-2}}{4x^5}$ (b) $(3x^4y^{-2})^{-2}$ (c) $\frac{2x^{-5}y^{-2}}{x^{-2}y^{-1}}$ (d) $\frac{x^{-2} - y^{-1}}{x^{-2} + y^{-1}}$

3.3 Exercise

1 Find the value of each of the following.

(a) 4^0 (b) $4^{\frac{1}{2}}$ (c) $4^{-\frac{1}{2}}$ (d) $27^{\frac{1}{3}}$

(e) $27^{-\frac{1}{3}}$ (f) 27^0 (g) -100^0 (h) $-100^{\frac{1}{2}}$

(i) $-100^{-\frac{1}{2}}$ (j) $(0.04)^0$ (k) $(0.04)^{-\frac{1}{2}}$ (l) $(0.04)^{\frac{1}{2}}$

2 Simplify.

(a) $25^{\frac{1}{2}}$ (b) $\sqrt{25}$ (c) $25^{-\frac{1}{2}}$ (d) 4^2

(e) $(\sqrt{4})^{-1}$ (f) 4^{-2} (g) $16^{\frac{1}{4}}$ (h) $\sqrt[4]{16}$

(i) $-\sqrt[4]{16}$ (j) $-27^{-\frac{1}{3}}$ (k) $-27^{-\frac{1}{3}}$ (l) $-\sqrt[3]{27}$

3 Find the value of each of the following.

(a) $32^{\frac{1}{5}}$ (b) $32^{\frac{2}{5}}$ (c) $32^{\frac{3}{5}}$

(d) $\sqrt[5]{32}$ (e) $\sqrt[5]{32^2}$ (f) $\sqrt[5]{32^3}$

4 Find the value of each of the following.

(a) $125^{-\frac{1}{3}}$ (b) $4^{\frac{3}{2}}$ (c) $100^{\frac{1}{2}}$ (d) $16^{-\frac{1}{4}}$

(e) $-27^{\frac{1}{3}}$ (f) $256^{\frac{1}{4}}$ (g) $8^{\frac{2}{3}}$ (h) $64^{\frac{2}{3}}$

(i) $4^{-\frac{3}{2}}$ (j) $-32^{\frac{1}{5}}$ (k) $125^{\frac{2}{3}}$ (l) $-27^{\frac{2}{3}}$

5 Write each of the following in a radical form.

(a) $3^3 = 27$ (b) $3^5 = 243$ (c) $(-1)^5 = -1$

(d) $2^4 = 16$ (e) $(-2)^5 = -32$ (f) $4^3 = 64$

(g) $5^4 = 625$ (h) $7^4 = 2401$ (i) $0.04 = (0.2)^2$

6 Which of the following represent rational numbers? Give reasons for your answers.

(a) $\sqrt[3]{-27}$ (b) $\sqrt{-4}$ (c) $(4)^{\frac{1}{2}}$

(d) 10^0 (e) $\sqrt[4]{-16}$ (f) $(-8)^{\frac{1}{3}}$

7 Simplify each of the following. Write with a single base.

(a) $m^3 \times m^2 \times m^{-1}$ (b) $x^4 \times x^5 \times x^0$ (c) $p^{-2} \times p^3 \times p^4$

(d) $2^3 \times 2^{-2} \times 2^0$ (e) $10^3 \times 10^{-1} \times 10^0$ (f) $(-3)^2(-3)^{-2}(-3)^0$

(g) $(3^{-2})^3$ (h) $(2^{-3})^2$ (i) $[(-a)^3]^{-2}$

(j) $\left(\frac{b}{a^{-2}}\right)^2$ (k) $\left(\frac{x^{-1}}{y}\right)^{-5}$ (l) $\left(\frac{a^2}{2^2}\right)^{-3}$

8 Simplify.

(a) $16^{\frac{1}{4}}$ (b) $-125^{\frac{1}{3}}$ (c) $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ (d) $\sqrt[4]{16}$

(e) $\left(\frac{25}{36}\right)^{\frac{1}{2}}$ (f) $-\sqrt[3]{\frac{27}{125}}$ (g) $16^{-\frac{5}{4}}$ (h) $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$

(i) $-1000^{\frac{1}{3}}$ (j) $\frac{1}{(27)^{\frac{2}{3}}}$ (k) $-125^{\frac{2}{3}}$ (l) $\sqrt[3]{8^2}$

(m) $-\sqrt[4]{16^{-2}}$ (n) $\left(\frac{32}{50}\right)^{\frac{1}{2}}$ (o) $16^{\frac{5}{4}}$ (p) $\sqrt[3]{\frac{1}{8}}$

(q) $\frac{1}{16^{-\frac{3}{4}}}$ (r) $-\sqrt[5]{243}$ (s) $\sqrt[5]{-243}$ (t) $-\sqrt[3]{-243}$

9 Evaluate each of the following and express your answer with denominator 1.

(a) $4^{-\frac{1}{2}}$ (b) $-4^{\frac{3}{2}}$ (c) $16^{-\frac{3}{4}}$ (d) $(\sqrt[4]{16})^{-1}$

(e) $25^{-\frac{3}{2}}$ (f) $\sqrt[3]{-8}$ (g) $(\sqrt{16})^{-1}$ (h) $8^{\frac{2}{3}}$

(i) $-\sqrt[5]{-32}$ (j) $(-\sqrt[3]{27})^{-1}$ (k) $\left(\frac{36}{49}\right)^{\frac{1}{2}}$ (l) $(-\sqrt{144})^2$

(m) $\left(\frac{100}{196}\right)^{\frac{1}{2}}$ (n) $(25^{\frac{1}{2}})^0$ (o) $(\sqrt[3]{-8})^3$

3.4 Exercise

1 Simplify.

(a) $49^{\frac{1}{2}} + 16^{\frac{3}{4}}$ (b) $27^{\frac{2}{3}} - 81^{\frac{1}{4}}$ (c) $16^{\frac{3}{4}} + 16^{\frac{3}{4}} - 81^{-\frac{3}{4}}$

(d) $128^{-\frac{2}{7}} - 16^{-0.25}$ (e) $16^{\frac{3}{2}} + 16^{0.75} + 5 - 27^{\frac{2}{3}}$

2 Simplify each of the following.

(a) $81^{\frac{1}{2}} + \sqrt[3]{8} - 32^{\frac{3}{5}} + 32^{-\frac{1}{5}}$ (b) $9^{\frac{1}{2}} - \sqrt[4]{16} + 81^{\frac{1}{4}} - 3(3^{-2})$

(c) $\left(\frac{1}{8}\right)^{\frac{1}{3}} - \sqrt[3]{\frac{27}{125}} + 2(16^{-\frac{3}{4}}) - (\sqrt{4.01})^0$ (d) $(-125)^{\frac{1}{3}} - 32^{-\frac{3}{5}} + \frac{1}{25^{\frac{1}{2}}} + \sqrt[3]{8^{-1}}$

3 Simplify each of the following.

(a) $\frac{3^{-1}}{2^{-1}}$ (b) $(9^{-\frac{1}{2}}) \div (16^{-\frac{3}{4}})$ (c) $3^{-2} \div 2^{-3}$

(d) $(3^{-2}) \div (27^{-\frac{2}{3}})$ (e) $\frac{3^{\frac{7}{12}}}{3^{\frac{1}{3}} \times 3^{\frac{1}{4}}}$ (f) $\frac{64^{\frac{5}{6}} \times 16^{\frac{3}{4}}}{27^{\frac{2}{3}}}$

4 Simplify.

(a) $\frac{2^{-1} + 3^{-1}}{6^{-1}}$ (b) $\frac{2^{-1}}{2^{-2} - 2^{-3}}$ (c) $\frac{2^{-1} - 2^{-3}}{3^{-1} - 2^{-1}}$

(d) $\frac{4^{-\frac{1}{2}} + 9^{-\frac{1}{2}}}{27^{-\frac{2}{3}}}$ (e) $\frac{8^{\frac{2}{3}} + 8^{\frac{2}{3}}}{8^{\frac{1}{3}} - 8^{\frac{2}{3}}}$ (f) $\frac{81^{\frac{1}{4}} + 81^{\frac{1}{4}}}{81^{\frac{3}{4}} - 81^{\frac{1}{4}}}$

5 Simplify.

(a) $\frac{x^{\frac{1}{2}} \times x^{\frac{2}{3}}}{x^{\frac{1}{4}}}$ (b) $\frac{x^{\frac{5}{6}} \times x^{\frac{2}{3}}}{x^{\frac{1}{2}}}$ (c) $(y^{\frac{1}{2}})^3 \div (4y^4)^{\frac{1}{2}}$

(d) $\left(\frac{\sqrt[4]{y^2}}{\sqrt{y}}\right)^{10}$ (e) $\left(\frac{x^3}{16}\right)^{\frac{1}{4}}\left(\frac{81^{\frac{3}{4}}}{x}\right)$ (f) $\frac{(x^2y^4)^{\frac{1}{2}}(x^4y^2)^{\frac{1}{2}}}{(x^{\frac{1}{2}}y^{\frac{1}{2}})^4}$

(g) $\sqrt[3]{\frac{\sqrt{x}\sqrt{x^3y}}{x^{\frac{4}{3}}}}$ (h) $(\sqrt[3]{x^{3n+1}})(\sqrt[3]{x^{n-1}})$ (i) $\left(\frac{a^5b^{-2}}{x^{-2}y^3}\right)^2\left(\frac{a^{-2}b^{-4}}{x^{\frac{1}{2}}y^{\frac{3}{2}}}\right)$

6 Simplify each of the following.

(a) $\left(\frac{a}{b}\right)^{-3}(ab^3)^2$ (b) $(x^k)^{-2}(-a^{-3})^k$ (c) $(x^n)^{-3} \div (x^{-n})^2$

(d) $\left(\frac{-b}{a}\right)^{-2}\left(\frac{a}{-b}\right)^2$ (e) $\frac{(ab^{-1})^3}{a^{-2}}\left(\frac{a}{b}\right)^{-3}$ (f) $\frac{(a^3)^{-k}(a^{-k})^2}{a^{-5k}}$

7 Find the value of each of the following if $a = 1$, $b = 2$, $k = 2$.

(a) ab^k (b) a^kb (c) $(ab)^k$ (d) $(ab)^{-k}$

(e) $(-ab)^k$ (f) $(a^{-1}b^{-2})^k$ (g) $(a^{-1}b^{-2})^{-k}$ (h) $(a^3b^2)^k$

8 Express each of the following with positive exponents.

(a) a^3b^{-2} (b) $\frac{3a^{-1}b}{c^{-1}}$ (c) $\frac{3ab^{-2}}{4m^{-2}}$ (d) $\frac{3^{-1}a^2b}{c^{-2}}$ (e) $\frac{(5a)^{-1}b}{c^{-2}}$

9 Each of the following represents a real number. What are the restrictions on the variables?

(a) $\sqrt{3x+1}$ (b) $\sqrt{2y-3}$ (c) $\sqrt{m^2-25}$ (d) $\sqrt[3]{3-p}$ (e) $\sqrt[4]{y+1}$

10 Find the smallest positive value of the variable so that each of the following represents an integer.

(a) $\sqrt{x+1}$ (b) $\sqrt{3p-1}$ (c) $\sqrt{\frac{y+5}{2}}$ (d) $\sqrt{\frac{2(x-5)}{3}}$ (e) $\sqrt{x^2-1}$

11 For $p = 2$, $k = -1$, which expression has the greater value?

A: $\frac{p^{-2k}p^{-k+2}}{(p^{-2})^k}$ B: $\frac{(p^k)^{-3}p^{-(1-2k)}}{(p^{-k})^3}$

Answers (3.2 - 3.4)

3.2 Exercise, page 90

1.a) $-\frac{1}{9}$ b) $\frac{1}{9}$ c) -1 d) 1 e) $\frac{4}{3}$ f) $\frac{1}{12}$ g) 9 h) $-\frac{1}{9}$ 2.a) $\frac{1}{8}$
b) $\frac{1}{4}$ c) $\frac{1}{25}$ d) -1 e) 9 f) 1 g) $\frac{2}{3}$ h) $\frac{8}{9}$ i) 32 j) $\frac{9}{4}$ 3.a) $\frac{1}{x^2}$
b) $\frac{1}{y^3}$ c) $\frac{1}{a^7}$ d) $\frac{1}{b^5}$ e) x^2 f) a g) c^4 h) $\frac{2}{x^4}$ i) $\frac{a}{b^5}$ j) $\frac{y^4}{x^3}$ k) $\frac{4}{a^2}$
l) $\frac{3y^2}{2x^4}$ 4.a) x^{-3} b) xy^{-4} c) $3ab^{-3}$ d) $4a^3$ e) $3a^2b$
f) $3(x+y)^2$ 5.a) a^2 b) $\frac{1}{m^6}$ c) a^{13} d) $\frac{1}{3a}$ e) x^6 f) $\frac{1}{xy^2}$ g) $\frac{1}{a^2}$
h) $\frac{1}{b^{12}}$ 6.a) $\frac{1}{100}$ b) 24 c) $\frac{17}{4}$ d) 36 e) 0 f) 6 7.a) -6 b) $\frac{1}{17}$
c) $\frac{26}{17}$ d) $-\frac{1}{9}$ 8.a) 2^{18-6n} b) 3^{2-5m} 9.a) 1 b) $\frac{16}{9}$ c) -1 d) $\frac{1}{3}$

3.3 Exercise, page 93

1.a) 1 b) 2 c) $\frac{1}{2}$ d) 3 e) $\frac{1}{3}$ f) 1 g) -1 h) -10 i) $-\frac{1}{10}$
j) 1 k) 5 l) $\frac{1}{5}$ 2.a) 5 b) 5 c) $\frac{1}{5}$ d) 16 e) $\frac{1}{2}$ f) $\frac{1}{16}$ g) 2 h) 2
i) -2 j) $-\frac{1}{3}$ k) -3 l) -3 3.a) 2 b) 4 c) 8 d) 2 e) 4 f) 8 4.a) $\frac{1}{5}$
b) 8 c) 10 d) $\frac{1}{2}$ e) -3 f) 4 g) 4 h) 16 i) $\frac{1}{8}$ j) -2 k) 25
l) -9 5.a) $3 = \sqrt[3]{27}$ b) $3 = \sqrt[5]{243}$ c) $-1 = \sqrt[5]{-1}$
d) $2 = \sqrt[4]{16}$ e) $-2 = \sqrt[5]{-32}$ f) $4 = \sqrt[3]{64}$ g) $5 = \sqrt[4]{625}$
h) $7 = \sqrt[4]{2401}$ i) $0.2 = \sqrt{0.04}$ 6.(a), (c), (d) 7.a) m^4
b) x^9 c) p^5 d) 2 e) 10^2 f) 1 g) $\frac{1}{3^6}$ h) $\frac{1}{2^6}$ i) $\frac{1}{a^6}$ j) a^4b^2 k) x^5y^5
l) $\frac{64}{a^6}$ 8.a) 2 b) -5 c) $\frac{1}{2}$ d) 2 e) $\frac{5}{6}$ f) $-\frac{3}{5}$ g) $\frac{1}{32}$ h) 2
i) -10 j) $\frac{1}{9}$ k) -25 l) 4 m) $-\frac{1}{4}$ n) $\frac{4}{5}$ o) 32 p) $\frac{1}{8}$ q) 8
r) -3 s) -3 t) 3 9.a) 2^{-1} b) -8 c) 8^{-1} d) 2^{-1} e) 125^{-1}
f) -2 g) 4^{-1} h) 4 i) 2 j) -3^{-1} k) 6×7^{-1} l) 144
m) 5×7^{-1} n) 1 o) -8

3.4 Exercise, page 95

1.a) 15 b) 6 c) $15\frac{26}{27}$ d) $-\frac{1}{4}$ e) 68 2.a) $3\frac{1}{2}$ b) $3\frac{2}{3}$
c) $-\frac{17}{20}$ d) $-\frac{177}{40}$ 3.a) $\frac{2}{3}$ b) $\frac{8}{3}$ c) $\frac{8}{9}$ d) 1 e) 1 f) $\frac{256}{9}$
4.a) 5 b) 4 c) $-\frac{9}{4}$ d) $\frac{15}{2}$ e) -4 f) $\frac{1}{4}$ 5.a) $x^{\frac{11}{12}}$ b) x
c) $\frac{1}{2y^{\frac{1}{2}}}$ d) 1 e) $\frac{27}{2x^{\frac{1}{4}}}$ f) xy g) $x^{\frac{1}{6}}y^{\frac{1}{6}}$ h) x^n i) $\frac{a^8x^{\frac{7}{2}}}{b^8y^{\frac{15}{2}}}$
6.a) $\frac{b^9}{a}$ b) $\frac{(-1)^k}{a^{3k}x^{2k}}$ c) $\frac{1}{x^p}$ d) $\frac{a^4}{b^4}$ e) a^2 f) 1 7.a) 4 b) 2 c) 4
d) $\frac{1}{4}$ e) 4 f) $\frac{1}{16}$ g) 16 h) 16 8.a) $\frac{a^3}{b^2}$ b) $\frac{3bc}{a}$ c) $\frac{3am^2}{4b^2}$
d) $\frac{a^2bc^2}{3}$ e) $\frac{bc^2}{5a}$ 9.a) $x \geqq -\frac{1}{3}$ b) $y \geqq \frac{3}{2}$ c) $|m| \geqq 5$
d) no restriction e) $y \geqq -1$ 10.a) 0 b) $\frac{1}{3}$ c) 3 d) 5 e) 1 11.A