Lesson 2.3B: Horizontal and Vertical Translations of Functions

Part A: Vertical Translations

Using Desmos, describe the transformations to the base graph in each case.

Graph a couple of equations at a time so that you can see the transformation from the base function.

a) $f(x) = x^2$ BASE FUNCTION

b) $g(x) = f(x) + 5$ graph moves up 5 units

c) $h(x) = f(x) - 3$ graph moves down 3 units

$x^2 + 5$

d) $f(x) = \sqrt{x}$ BASE FUNCTION

e) $g(x) = f(x) + 4$ graph moves up 4 units

f) $h(x) = f(x) - 2$ graph moves down 2 units

$\sqrt{x} + 4$

Think about the Domain and Range as you graph....

Try graphing the base function along with each of these:

g) $m(x) = \frac{1}{x} + 3$ Base Function: graph moves

h) $n(x) = x^3 - 5$ Base Function: graph moves

General Result

g(x) = f(x) + c is a vertical translation of the graph of f(x).

If c > 0, the graph of *f(x)* moves UP c units .

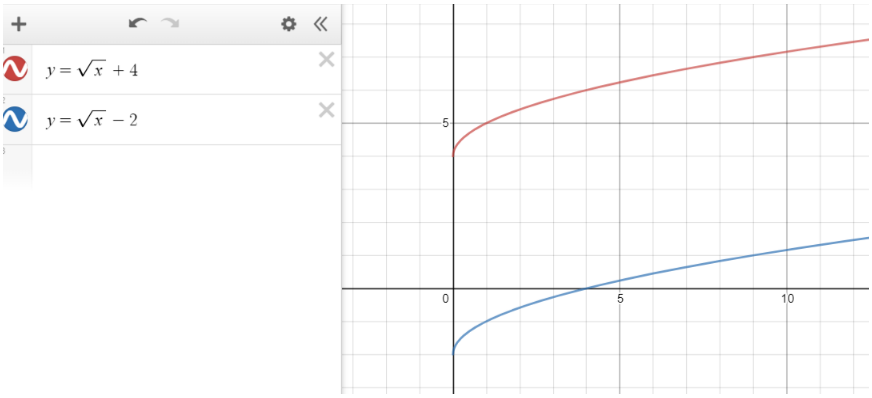
If c < 0, the graph of *f(x)* moves DOWN c units .

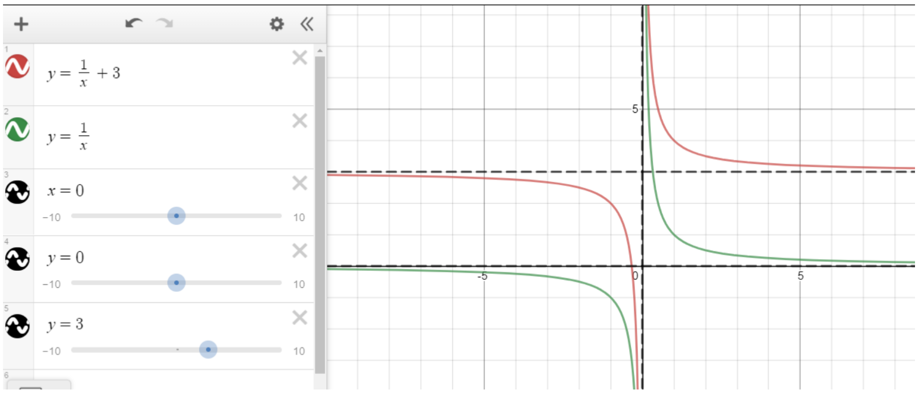
The domain does not change . The range can change .

x-values are not affected . y-values are affected .

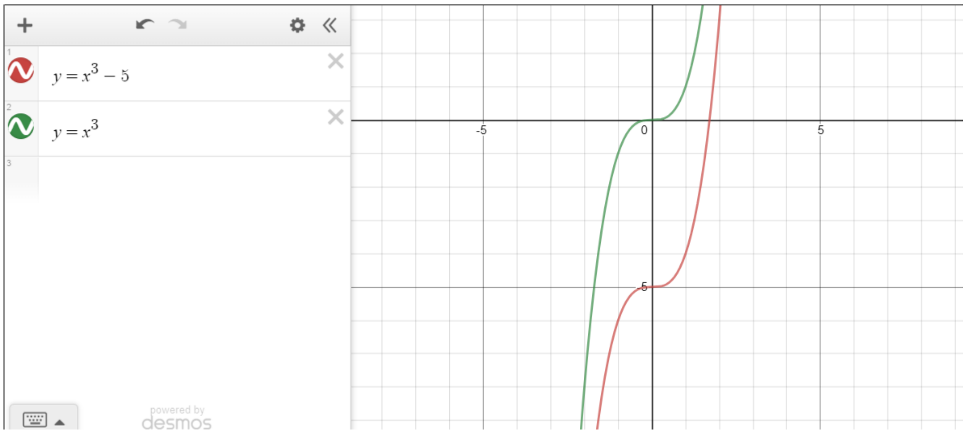
c is OUTSIDE of the function so no x-values change.


$y = x^2 + 5$
$y = x^2 - 3$
-5
0
5
5
powered by desmos


$y = \sqrt{x} + 4$
$y = \sqrt{x} - 2$
5
0
5
10



$y = \frac{1}{x} + 3$
$y = \frac{1}{x}$
$x = 0$
$y = 0$
$y = 3$



$y = x^3 - 5$
$y = x^3$

Part B - Horizontal Translations

Graph the following using Desmos and compare to the base function.

1. Graph $f(x)=x^2$ and the equations below. Describe the transformations.

   a) $g(x)=f(x+4)$ $= (x+4)^2$ left 4 units

   b) $h(x)=f(x-2)$ right 2 units

2. Graph $f(x)=\sqrt{x}$ and the equations below. Describe the transformations.

   a) $g(x)=f(x+1)$ $\sqrt{x+1}$ left 1 unit

   b) $h(x)=f(x-4)$ right 4 units.

General Result

g(x) = f(x - d) is a horizontal translation of the graph of f(x). shift left/right

opposite

If d > 0, the graph of *f(x)* moves RIGHT d units.

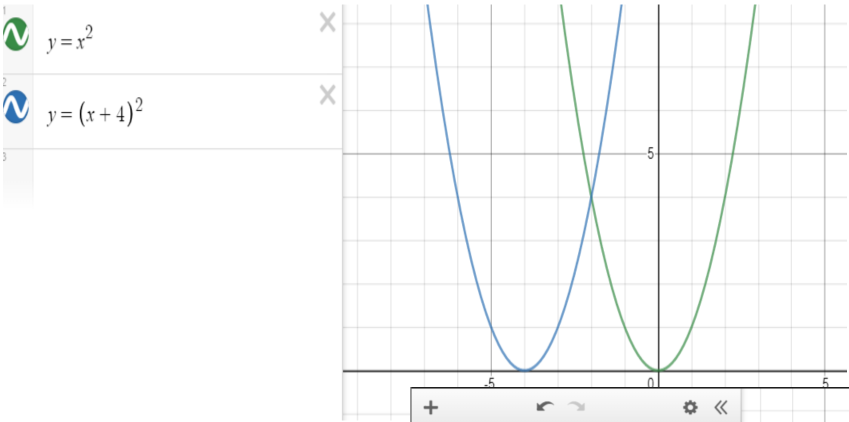
If d < 0, the graph of *f(x)* moves LEFT d units.

The domain can change. The range does not change.

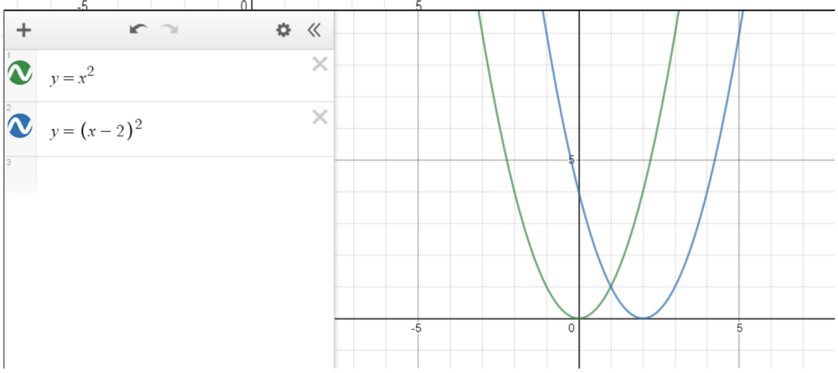
x-values are affected. y-values are not affected.

d is INSIDE the function so no y-values change.

This transformation is the opposite of what you think because the x-coord must compensate for its change in order for the y-coord to stay the same.

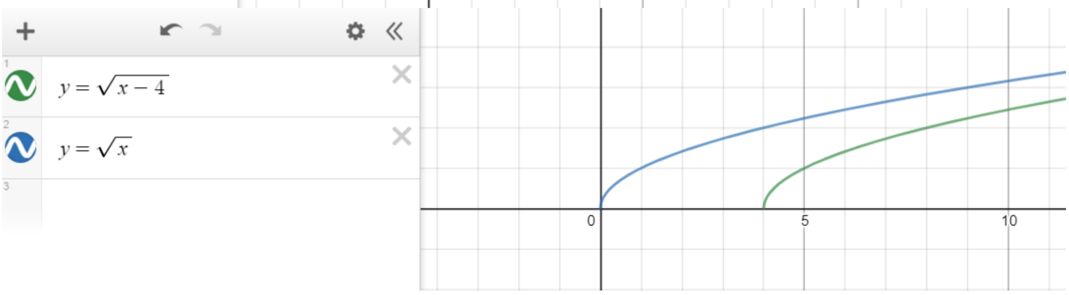


y = x^2
y = (x + 4)^2
5
-5
0
5



y = x^2
y = (x − 2)^2
5
-5
0
5

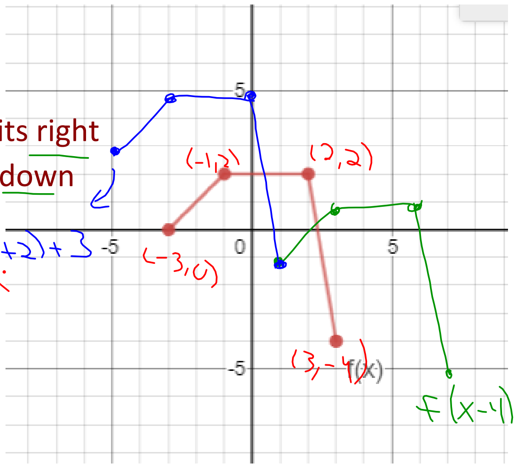

$y=\sqrt{x+1}$
$y=\sqrt{x}$
0
5
10


$y=\sqrt{x-4}$
$y=\sqrt{x}$
0
5
10

Ex. 1: Given the graph of $f(x)$ shown below, graph:

Graphing Process
- Plot 3 to 5 base points from the parent function.
- Transform these points in order to create the graph.

OR
- Use mapping notation to find the coordinates of the transformed points.



a) $g(x) = f(x-4)-1$
- horizontal translation 4 units right
- vertical translation 1 units down

Solution

Mapping Notation:
$(x, y) \to (x+4, y-1)$
$(-3,0) \to (1,-1)$
$(-1,2) \to (3,1)$
$(2,2) \to (6,1)$

b) $h(x) = f(x+2)+3$
- h.t. 2 units left
- v.t. 3 units up

Mapping Notation
$(x, y) \to (x+d, y+c)$

Solution

$(x,y) \to (x-2, y+3)$

Ex. 2: Find the equation of $g(x) = f(x+1)-3$ if:

a) $f(x) = x^2$ Base.
$g(x) = (x+1)^2 - 3$

b) $f(x) = x^3$ Base.
$g(x) = (x+1)^3 - 3$

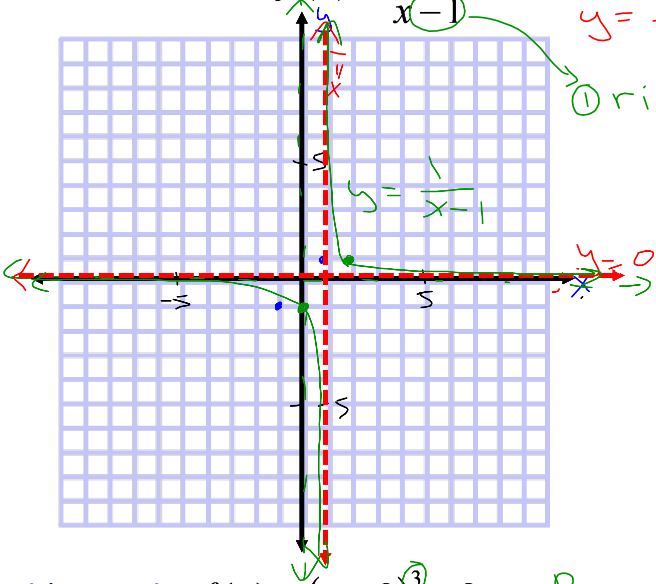
c) $f(x) = \sqrt{x}$ Base
$g(x) = \sqrt{x+1} - 3$

d) $f(x) = \frac{1}{x}$ Base
$g(x) = \frac{1}{x+1} - 3$

Ex. 3: a) Graph $f(x)=\dfrac{1}{x-1}$.

Base $y=\frac{1}{x}$

① right 1



b) State the domain and range.

$D:\{x\in R \mid x\neq 0\}$
$R:\{y\in R \mid y\neq 0\}$ Base

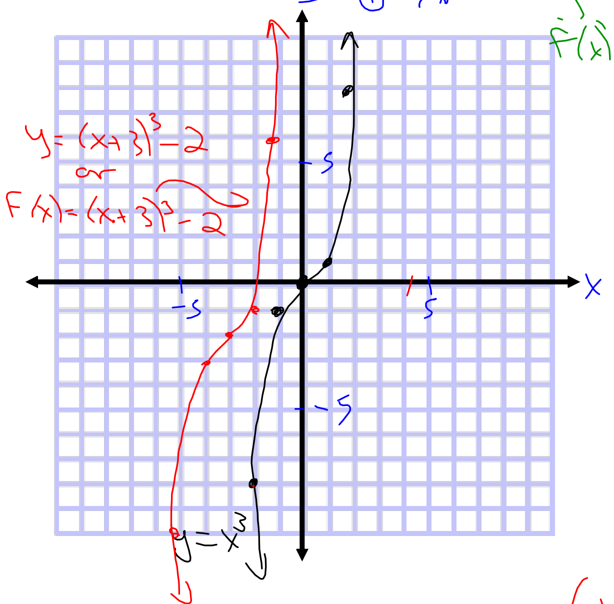
$D=\{x\in R, x\neq 1\}$

$R=\{y\in R, y\neq 0\}$

Solution

b) Graph $f(x)=(x+3)^3-2$.

Base $y=x^3$, $f(x)=x^3$



b) State the domain and range.

$D:\{x\in R\}$
$R:\{y\in R\}$ Base

① shift left 3

② down 2

$D=\{x\in R\}$

$R=\{y\in R\}$

Mapping Notation

$(x,y)\rightarrow(x-3,y-2)$

$(-2,-8)\rightarrow(-5,-10)$

$(-1,-1)\rightarrow(-4,-3)$

Solution

HOMEWORK
p. 101 #C2, C3, 3c, 5b, 6eij, 8ad, 12c, 13c, 14c

+ Extra Practice Sheet 2.3B