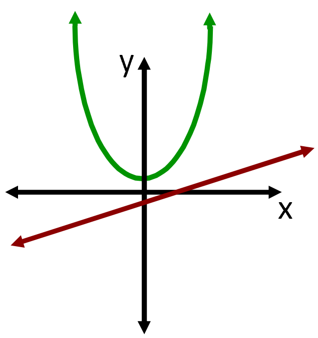
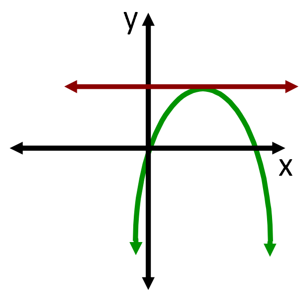
1.8 Solving Linear and Quadratic Systems

A <u>system</u> of equations consists of <u>two or more</u> equations. If the graphs in the system are linear (degree 1) and quadratic (degree 2), the system could have <u>no solution</u>, <u>one solution</u>, or <u>two solutions</u>.

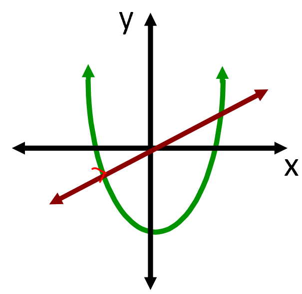
No Solution



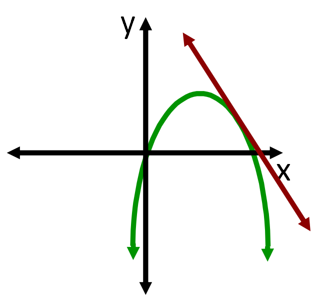
One Solution



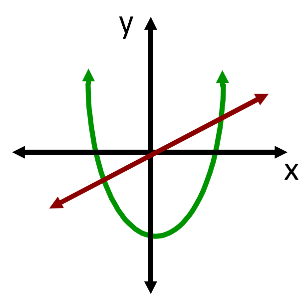
Two Solutions



<u>Tangent</u> - A line that intersects a curve at <u>one</u> point and has the same slope as the curve at that point.



<u>Secant</u> - A line that intersects a curve at <u>two</u> distinct points.



Process for solving a linear-quadratic system algebraically:

1. Isolate one variable from the linear equation.
2. Sub into the quadratic.
3. Solve for the remaining variable.
4. Sub answer(s) back into the linear equation to find the coordinate(s) of intersection, if they exist.

Ex. 1 Solve the system.

① $y = x^2 - 3$

② $2x + y = -3$

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From ②

$y = -3 - 2x$

Sub into ①

$-3 - 2x = x^2 - 3$

$0 = x^2 + 2x$

$0 = x(x+2)$

$x = 0$ $\quad$ $x = -2$

Sub each x back into ②

$2(0) + y = -3$

$y = -3$

$\therefore (0, -3)$

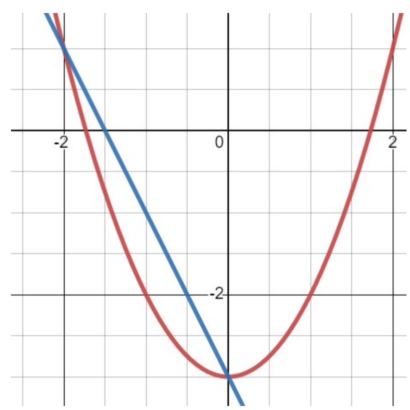
$2(-2) + y = -3$

$-4 + y = -3$

$y = 1$

$(-2, 1)$

<u>Graphically</u>



x

Ex. 2 Find the coordinates of the point of intersection between the parabola $y-4 = -(x + 1)^2$ and the line $y = 3x + 13$.

① $y-4=-(x+1)^2$

② $y = 3x+13$

Sub ② into ①

$3x+13-4=-(x+1)^2$

$3x+9 = -(x^2+2x+1)$

$0 = -x^2-5x-10$

$x^2+5x+10=0$

$a=1 \quad b=5 \quad c=10$

Use Quad. Form.

M 10

A 5

N ?

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2-4(1)(10)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25-40}}{2}$$

Negative!

∴ No solutions

Ex. 3 If a line with a slope of 4 has one point of intersection with the quadratic function $y=\frac{1}{2}x^2 + 2x - 8$, what is the y-intercept of the line? Write the equation of the line in slope y-intercept form.

Discrimant!
$D = 0$

① $y=4x+b$

② $y=\frac{1}{2}x^2+2x-8$

Sub ① into ②

$4x+b=\frac{1}{2}x^2+2x-8$

$0=\frac{1}{2}x^2-2x-8-b$

$D=b^2-4ac$

$D=0$ for one solution

$a=\frac{1}{2}$ $b=-2$ $c=-8-b$

$0=(-2)^2-4(\frac{1}{2})(-8-b)$

$0=4-2(-8-b)$

$0=4+16+2b$

$0=20+2b$

$-20=2b$

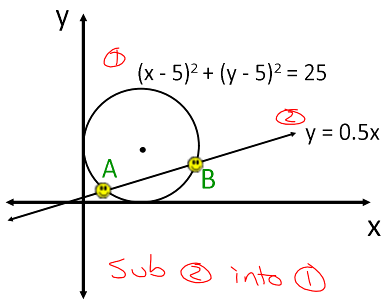
$\boxed{-10=b}$

$\therefore$ y-int is -10

Ex. 3 Find the length of chord AB rounded to two decimal places.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Length of a line segment formula.



Sub ② into ①

$$(x-5)^2 + (0.5x-5)^2 = 25$$

$$x^2 - 10x + 25 + 0.25x^2 - 5x + 25 = 25$$

$$1.25x^2 - 15x + 25 = 0$$

Solve via Quad. Form. or factoring

$a = 1.25$
$b = -15$
$c = 25$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{15 \pm \sqrt{(-15)^2 - 4(1.25)(25)}}{2(1.25)}$$

$$= \frac{15 \pm \sqrt{100}}{2.5}$$

$$= \frac{15 \pm 10}{2.5}$$

$= \frac{15+10}{2.5}$ $= 10$

$= \frac{15-10}{2.5}$ $= 2$

$$\frac{5}{4}x^2 - 15x + 25 = 0$$

$$5x^2 - 60x + 100 = 0 \quad (\times 4)$$

$$5(x^2 - 12x + 20) = 0$$

$$5(x-2)(x-10) = 0$$

$x = 2$ $x = 10$

$x = 2$ & $x = 10$ are x-values of our intersections!

② $y = 0.5x$

for $x = 2$
$y = 0.5(2)$
$= 1$
$(2, 1)$

for $x = 10$
$y = 0.5(10)$
$= 5$
$(10, 5)$

Now we need distance between (2,1) & (10,5)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(10-2)^2 + (5-1)^2}$$

$$= \sqrt{64 + 16}$$

$$= \sqrt{80}$$

$$\doteq 8.94$$

$\therefore$ The chord length is approx 8.94 units.

Homework

p. 67 # C2, 1ac, 3ac, 5ab, 7, 10, 15, 19