Lesson 1.7A: Determining a Quadratic Equation Given Its Roots

Recall: Quadratics can be represented in a number of different forms:

Note: all forms give 'a' value
- stretch value
- direction of opening

Standard Form

$$f(x) = ax^2 + bx + c$$

What do each of these forms tell you about the quadratic function?

complete the square → Vertex Form

factor → Factored Form

partial factor → Partially Factored Form

Vertex Form	Factored Form	Partially Factored Form
$f(x) = a(x-h)^2 + k$ - Vertex (h, k) when it occurs Max/Min value	$f(x) = a(x-r)(x-s)$ - x-ints: r & s - find axis of symm. $x = \frac{r+s}{2}$ - sub into equation to find max/min	$f(x) = ax(x-s) + t$ - gives two points on the graph, same y-co-ordinate, t $(0, t)$ (s, t) - find axis of symm $x = \frac{0+s}{2}$

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Ex. 1 Find the equation, in factored form, for a <u>family</u> of quadratic functions that has zeros at x = 4 and x = -3. Sketch three possible members of this family.

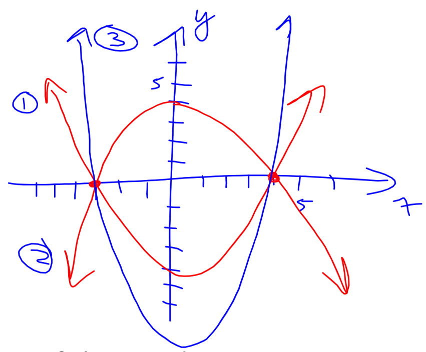
Factored form → $y=a(x-r)(x-s)$

$f(x)=a(x-4)(x+3)$

① $f(x)=(x-4)(x+3)$

② $f(x)=-(x-4)(x+3)$

③ $f(x)=2(x-4)(x+3)$



Ex. 2 Algebraically determine the equation of the quadratic function, in standard form, having only one x-intercept, at x = 2 (double root), and containing the point (3,10).

Factored Form

$f(x)=a(x-2)(x-2)$

$=a(x-2)^2$

$f(3)=10$

$10=a(3-2)^2$

$10=a(1)^2$

$10=a$

Vertex (Vertex @ (2,0))

$f(x)=a(x-2)^2+0$

$=a(x-2)^2$

Standard Form

$f(x)=10(x-2)^2$

$=10(x-2)(x-2)$

$=10(x^2-4x+4)$

$f(x)=10x^2-40x+40$

Ex. 3 Algebraically determine an equation, in factored form, of the parabola that has x-intercepts $3+\sqrt{7}$ and $3-\sqrt{7}$, and that passes through the point (-5,3).

$f(x)=a(x-(3+\sqrt{7}))(x-(3-\sqrt{7}))$

$=a(x-3-\sqrt{7})(x-3+\sqrt{7})$

sub in (-5,3)

$f(-5)=3$

$3=a(-5-3-\sqrt{7})(-5-3+\sqrt{7})$

$3=a(-8-\sqrt{7})(-8+\sqrt{7})$

$3=a(64-\cancel{8\sqrt{7}}+\cancel{8\sqrt{7}}-7)$

$3=a(57)$

$\frac{3}{57}=a$

$\frac{1}{19}=a$

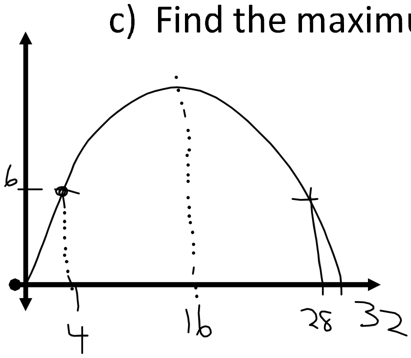
$\therefore f(x)=\frac{1}{19}(x-3-\sqrt{7})(x-3+\sqrt{7})$

Ex. 4 The parabolic opening to a tunnel is 32 m wide measured from side to side along the ground. At points 4 m from each side, the tunnel entrance is 6 m high.

a) Sketch a diagram of the given information.

b) Determine the equation of the function that models the opening to the tunnel.

c) Find the maximum height of the tunnel, to the nearest tenth.



b) $f(x) = a(x - 0)(x - 32)$

$= ax(x - 32)$

Sub in (4,6)

$6 = a(4)(4 - 32)$

$6 = a(4)(-28)$

$6 = a(-112)$

$-\frac{6}{112} = a$

$-\frac{3}{56} = a$

$\therefore f(x) = -\frac{3}{56}x(x - 32)$

c) $f(16) = -\frac{3}{56}(16)(16 - 32)$

$= -\frac{3}{56}(16)(-16)$

$\doteq 13.7$ $\therefore$ The max height is 13.7m

HOMEWORK

p. 57 #C3, 1a, 3ac, 4ac, 5ac, 6a, 8a, 11c, 15