1.6 Solve Quadratic Equations

Recall: Solving a quadratic equation means finding the value of the roots, zeros or x-intercepts. You are finding where the function, f(x) is zero.

A. Solve by Factoring

Graphs on next page...

How do you find zeros?
1. Set f(x) = 0.
2. Factor.
3. Set each factor = 0 and solve for x.

Ex. 1 Solve each of the following:

a) $f(x) = (x-3)(x+4)$

Find Zeroes $f(x) = 0$

$0 = (x-3)(x+4)$

$x-3=0$ $\quad x+4=0$

$x=3$ $\quad x=-4$

b) $f(x) = x^2 + 7x - 30$

$0 = x^2+7x-30$

$= (x-3)(x+10)$

$x=3$ $\quad x=-10$

c) $f(x) = 4x^2 - 9$

$0 = (2x+3)(2x-3)$

$2x+3=0$ $\quad 2x-3=0$

$2x=-3$ $\quad x=\frac{3}{2}$

$x=-\frac{3}{2}$

d) $f(x) = 3x^2 + 12x$

$0 = 3x(x+4)$

$x=0$ $\quad x=-4$

e) Find the vertex of d)

Find Axis of symm.

$x = \frac{0+(-4)}{2}$

$= -2$

$f(-2) = 3(-2)(-2+4)$

$= -6(2)$

$= -12$

$\therefore V(-2, -12)$

B. Solve from Vertex Form

Ex. 1 Solve each of the following:

1. Set y = 0.
2. Isolate for x

$f(x) = 2(x-3)^2 - 8$

$8 = 2(x-3)^2$

$4 = (x-3)^2$

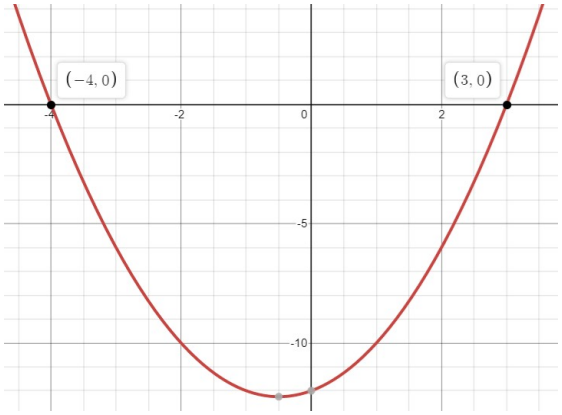
$\pm\sqrt{4} = x-3$

$\pm 2 + 3 = x$

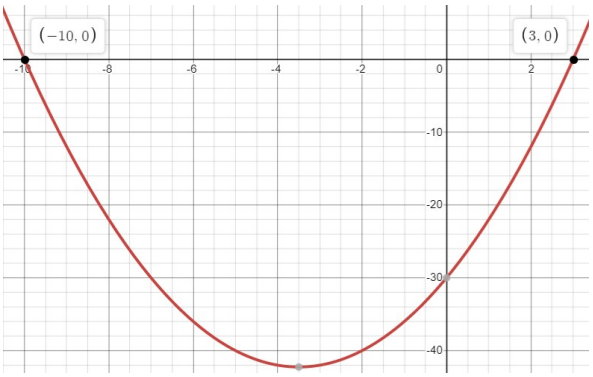
$x = 2+3 = 5$

$x = -2+3 = 1$

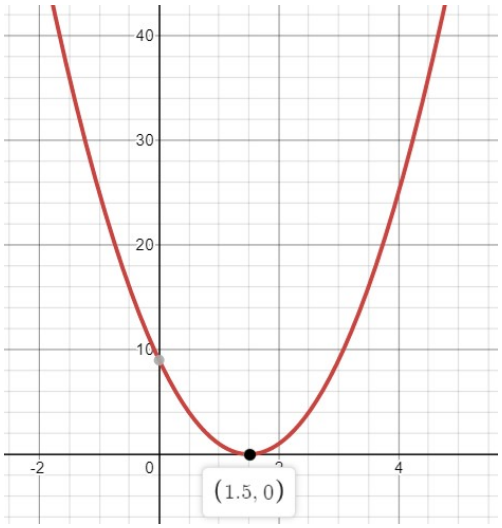
1. a) $f(x) = (x-3)(x+4)$



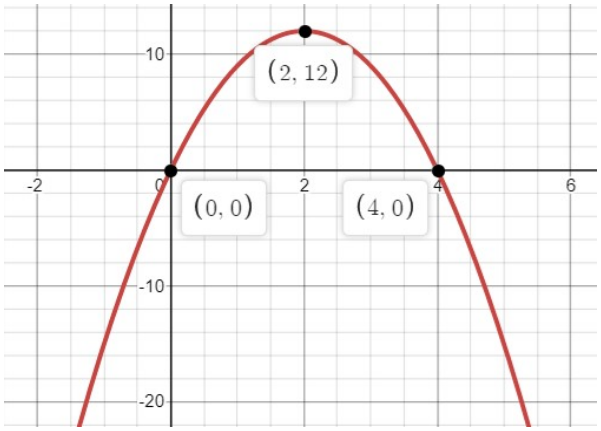
1. b) $f(x) = x^2 + 7x - 30$



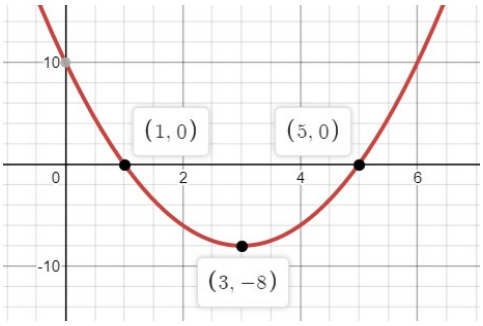
1. c) $f(x) = 4x^2 - 12x + 9$



1. d) $f(x) = -3x^2 + 12x$



2. a) $f(x) = 2(x-3)^2 - 8$



C. Solve using the Quadratic Formula

Exact answers only!!!

Recall:

The quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ex. 1 Solve. Give exact answers only.

a) $3x^2 + 4x - 2 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-4 \pm \sqrt{4^2 - 4(3)(-2)}}{2(3)}$

$= \dfrac{-4 \pm \sqrt{16 + 24}}{6}$

$= \dfrac{-4 \pm \sqrt{40}}{6}$

$= \dfrac{-4 \pm 2\sqrt{10}}{6}$

$= \dfrac{-2 \pm \sqrt{10}}{3}$

b) $5x^2 - 3x + 2 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{3 \pm \sqrt{(-3)^2 - 4(5)(2)}}{2(5)}$

$= \dfrac{3 \pm \sqrt{9 - 40}}{10}$

$= \dfrac{3 \pm \sqrt{-31}}{10}$

∴ No solutions!
- Zeros

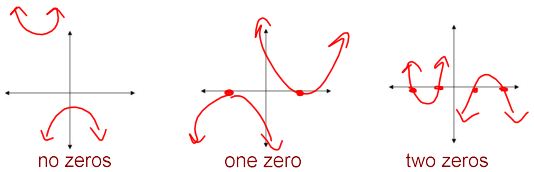
We can determine the number of roots by looking under the radical sign

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is known as the **Discriminant** $b^2 - 4ac$

Quadratics can have no zeros, 1 zero, or 2 zeros.

Sketch an example of each scenario:



- If $b^2 - 4ac > 0$ then there is two real roots
- If $b^2 - 4ac = 0$ then there is one real root
- If $b^2 - 4ac < 0$ then there is no real roots

Ex. 2 For each quadratic equation, determine the number of roots.

a) $2x^2 - x + 5 = 0$

$D = b^2 - 4ac$

$= (-1)^2 - 4(2)(5)$

$= 1 - 40$

$= -39$

$\therefore$ No roots

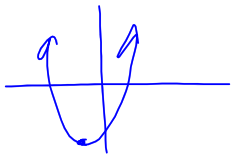
b) $4(x + 1)^2 - 7 = 0$

Vertex form!

$V(-1, -7)$

Opens UP

$\therefore$ 2 zeroes



c) $(x - 6)^2 = 0$

$V(-6, 0)$

On the x-axis!

$\therefore$ One root

d) $2x^2 + 8x + 8 = 0$

$D = b^2 - 4ac$

$= 8^2 - 4(2)(8)$

$= 64 - 64$

$= 0$

$\therefore$ One root

Homework
p. 49 # C1cdf, 3bdf, 5c, 6abc, 7, 12, 13a