Lesson 1.5A Working With Radicals

$\sqrt[n]{m}$

index

radicand

refers to some irrational numbers (radicand not a perfect square)

Properties of Radicals:

1) Multiplication

$\sqrt{4 \cdot 25}$
$= \sqrt{100}$
$= 10$

$\sqrt{4} \cdot \sqrt{25}$
$= 2 \cdot 5$
$= 10$

In general: $\sqrt{b} \bullet \sqrt{d}$
$= \sqrt{bd}$

$a\sqrt{b} \bullet c\sqrt{d}$
$= ac\sqrt{bd}$

2) Division

$\sqrt{\frac{81}{9}}$
$= \sqrt{9}$
$= 3$

$\frac{\sqrt{81}}{\sqrt{9}}$
$= \frac{9}{3}$
$= 3$

In general: $\frac{\sqrt{b}}{\sqrt{d}}$
$= \sqrt{\frac{b}{d}}$

$\frac{a\sqrt{b}}{c\sqrt{d}}$
$= \frac{a}{c}\sqrt{\frac{b}{d}}$

3) Squaring

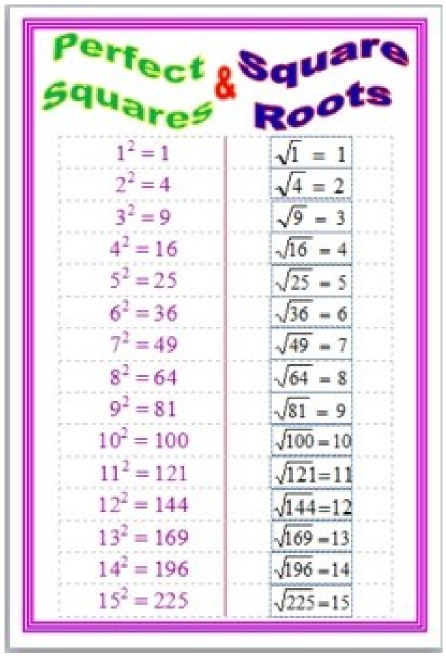
$\left(\sqrt{b}\right)^2$
$= b$

$= \sqrt{b} \cdot \sqrt{b}$
$= \sqrt{b \cdot b}$
$= \sqrt{b^2}$
$= b$

$\left(a\sqrt{b}\right)^2$
$= (a\sqrt{b})(a\sqrt{b})$
$= a^2(\sqrt{b})^2$
$= a^2 b$

In general:

$\left(a\sqrt{b}\right)^m$
$= a^m \sqrt{b^m}$



Perfect Squares & Square Roots

$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$
$11^2 = 121$	$\sqrt{121} = 11$
$12^2 = 144$	$\sqrt{144} = 12$
$13^2 = 169$	$\sqrt{169} = 13$
$14^2 = 196$	$\sqrt{196} = 14$
$15^2 = 225$	$\sqrt{225} = 15$

Radicals can be: **Entire** $\sqrt{n}$ or **Mixed** $a\sqrt{b}$

Sometimes entire radicals can be changed to mixed radicals by simplifying.

Ex. 1 Change Entire Radicals to Mixed Radicals

a) $\sqrt{27}$

$= \sqrt{9 \cdot 3}$

$= \sqrt{9}\sqrt{3}$

$= 3\sqrt{3}$

b) $\sqrt{48}$

$= \sqrt{16 \cdot 3}$

$= \sqrt{16} \cdot \sqrt{3}$

$= 4\sqrt{3}$

c) $\sqrt{500}$

$= \sqrt{100} \cdot \sqrt{5}$

$= 10\sqrt{5}$

d) $\sqrt{180}$

$= \sqrt{36 \cdot 5}$

$= 6\sqrt{5}$

A radical is in <u>simplest form</u> if:

1. The radical has no perfect square factors other than 1 in the radicand.
2. There are no fractions under a $\sqrt{}$. $\sqrt{\frac{1}{6}}$
3. There are no $\sqrt{}$ in the denominator. $\frac{1}{\sqrt{2}}$

Multiplying and Dividing Radicals:

Ex. 2 Simplify.

a) $\sqrt{5} \bullet \sqrt{7}$

$= \sqrt{5 \cdot 7}$

$= \sqrt{35}$

b) $3\sqrt{6} \bullet \sqrt{2}$

$= 3\sqrt{12}$

$= 3 \cdot \sqrt{4}\sqrt{3}$

$= 3 \cdot 2\sqrt{3}$

$= 6\sqrt{3}$

c) $(5\sqrt{6})(2\sqrt{8})$

$= 10\sqrt{48}$

$= 10\sqrt{16 \cdot 3}$

$= 40\sqrt{3}$

d) $(2\sqrt{6})(3\sqrt{2})(5\sqrt{6})$

$= 2 \cdot 3 \cdot 5 \cdot \sqrt{\underbrace{6 \cdot 2 \cdot 6}}$

$= 30 \cdot 6\sqrt{2}$

$= 180\sqrt{2}$

$\sqrt{72}$

$= \sqrt{36 \cdot 2}$

$= 6\sqrt{2}$

e) $\sqrt{3}(\sqrt{6} + 5)$

$= \sqrt{18} + 5\sqrt{3}$

$= \sqrt{9 \cdot 2} + 5\sqrt{3}$

$= 3\sqrt{2} + 5\sqrt{3}$

f) $\dfrac{\sqrt{18}}{\sqrt{3}} = \sqrt{\dfrac{18}{3}}$

$= \sqrt{6}$

g) $\dfrac{15\sqrt{7}}{3\sqrt{4}}$

$= \dfrac{5\sqrt{7}}{1 \cdot 2}$

$= \dfrac{5\sqrt{7}}{2}$

h) $\dfrac{5\sqrt{12}}{\sqrt{8}} = \dfrac{5 \cdot \sqrt{4 \cdot 3}}{\sqrt{4 \cdot 2}}$

$= \dfrac{10\sqrt{3}}{2\sqrt{2}}$

$= \dfrac{5\sqrt{3}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{5\sqrt{6}}{2}$

Rationalizing the Denominator

Multiply both the numerator and the denominator by the radical in the denominator.

Squaring the radical will eliminate it from the denominator.

i) $\dfrac{\overset{3}{\cancel{12}}\sqrt{27}}{\underset{1}{\cancel{4}}\sqrt{45}}$

$= \dfrac{3\sqrt{9 \cdot 3}}{\sqrt{9 \cdot 5}}$

$= \dfrac{3\sqrt{3}}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$

$= \dfrac{3\sqrt{15}}{5}$

Adding and Subtracting Radicals:

Algebra: Collect like terms.

Like Terms

Same variables, same exponents

Example: $2x^2, 5x^2$

Counter-example: $3x, 4x^2$

Radicals: Collect like radicals.

Like Radicals

Same index, same radicand

Example: $\sqrt{3}, 2\sqrt{3}$

Counter-example: $2\sqrt{3}, 5\sqrt{2}$

Ex. 3 Are the following radicals like or unlike?

a) $2\sqrt{3}, -3\sqrt{3}, 4\sqrt{3}$

LIKE

b) $\sqrt{4}, \sqrt{2}, \sqrt{3}$

UNLIKE

c) $\sqrt{8}, \sqrt{2}, \sqrt{32}$

$2\sqrt{2}$ $\sqrt{2}$ $4\sqrt{2}$

LIKE

d) $\sqrt[3]{3}, \sqrt{3}, \sqrt[4]{3}$

UNLIKE

Ex. 4 Add or Subtract.

a) $\sqrt{27}+\sqrt{20}-\sqrt{12}+\sqrt{45}$

$= 3\sqrt{3}+2\sqrt{5}-2\sqrt{3}+3\sqrt{5}$

$= \sqrt{3}+5\sqrt{5}$

b) $7\sqrt{2}-6\sqrt{63}-\sqrt{28}+5\sqrt{18}$

$= 7\sqrt{2}-18\sqrt{7}-2\sqrt{7}+15\sqrt{2}$

$= 22\sqrt{2}-20\sqrt{7}$

Homework

p. 39 # 1-3, 4bdf, 5bdf, 6bcde, 8ad, 9acd, 11, 13, 14, 16c

$$\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

You CANNOT split up the radical across a + or – sign.

$$\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$$
$$\sqrt{x-y} \neq \sqrt{x} - \sqrt{y}$$