1.4B Partial Factoring
~Max or Min of a Quadratic Function

Recall that to find the vertex we have:
- completed the square (time consuming)
- factored (vertex falls halfway between the zeros) (gr.10 and later in the unit)

And now for something sort of brand new...

Finding the Vertex by Partial Factoring

Partial Factoring involves finding two points on the parabola that have the same y-coordinate.

$f(x) = 3x^2 - 24x + 3$

What is the y-intercept? 3

it is the "c" value from standard form

(0 , 3)

is there another x-value with the same y-value?

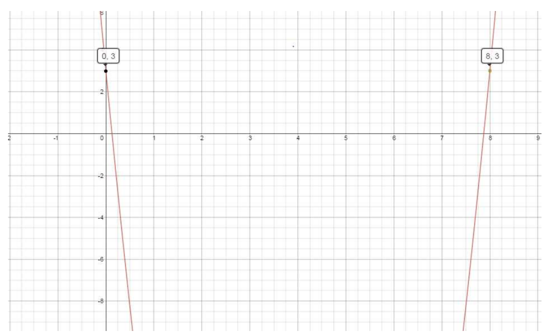
- need to have $3x^2 - 24x = 0$

Partially Factored Form:

$f(x) = 3x(x-8) + 3$

$(0,3)$ $(8,3)$

Graphically this is what is going on:



$f(x) = 3x^2 - 24x + 3$
$= 3x(x-8) + 3$
$f(0) = 3$
$f(8) = 3$

Now, determine the the axis of symmetry and the vertex.

$x = \frac{0+8}{2}$
$= 4$

Vertex?

Sub in $x = 4$

$y = 3x(x-8) + 3$
$= 3(4)(4-8) + 3$
$= 12(-4) + 3$
$= -48 + 3$
$= -45$

$\therefore$ Vertex $(4, -45)$

Partial Factored Form: $f(x) = ax(x - s) + t$

Process:

- From standard form, factor ax from the first two terms.
- Set x = 0, then y = t. (0, t) is the y-intercept.
- Set x = s, then y = t. (s, t) is a symmetrical point to the y-intercept.
- Determine the axis of symmetry.
- Determine the y-cordinate of the vertex.

The symmetrical points are NOT the Zeros!

Ex. 1 Use partial factoring to determine the vertex.

a) $f(x) = 2x^2 + 10x + 1$

$= 2x(x+5) + 1$

Pt $(0, 1)$

Pt $(-5, 1)$

$\underline{\text{Axis}}$

$x = \frac{0+(-5)}{2}$

$= -\frac{5}{2}$

$f\left(-\frac{5}{2}\right) = 2\left(-\frac{5}{2}\right)\left(-\frac{5}{2}+5\right) + 1$

$= -5\left(\frac{5}{2}\right) + 1$

$= -\frac{23}{2}$

$\therefore V\left(-\frac{5}{2}, -\frac{23}{2}\right)$

b) $f(x) = -2x^2 + 8x - 13$

$= -2x(x-4) - 13$

$x = 0, 4$

$\underline{\text{Axis}}$

$x = \frac{0+4}{2}$

$= 2$

$f(2) = -2(4) + 8(2) - 13$

$= -5$

$V(2, -5)$

c) $f(x) = -x^2 + 5x - 3$

$f(x) = -x(x-5) - 3$

$x = 0$ & 5

$\underline{\text{Axis}}$

$x = \frac{0+5}{2}$

$= \frac{5}{2}$

$f\left(\frac{5}{2}\right) = -\frac{5}{2}\left(\frac{5}{2} - 5\right) - 3$

$= -\frac{5}{2}\left(-\frac{5}{2}\right) - 3$

$= \frac{25}{4} - 3$

$= \frac{13}{4}$

$\therefore V\left(\frac{5}{2}, \frac{13}{4}\right)$

$f(x) = 5x^2 - 2x + 1$

$f(x) = 5x\left(x - \frac{2}{5}\right) + 1$

$x = 0$ & $\frac{2}{5}$

$\underline{\text{Axis}}$

$x = \frac{0 + \frac{2}{5}}{2}$

$= \frac{2}{10}$

$= \frac{1}{5}$

$f(x) = 5x\left(x - \frac{2}{5}\right) + 1$

$f\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)\left(\frac{1}{5} - \frac{2}{5}\right) + 1$

$= \left(-\frac{1}{5}\right) + 1$

$= \frac{4}{5}$

$\therefore V\left(\frac{1}{5}, \frac{4}{5}\right)$

Homework
p. 31 #3

Handout 1.4B

Find vertex by either completing the square or partial factoring - your choice