Lesson 1.4A: Completing the Square

~ Max or Min of a Quadratic Function

What do you remember about quadratics?

Standard Form	Vertex Form	Factored Form
$f(x)=ax^2+bx+c$	$f(x)=a(x-h)^2+k$	$f(x)=a(x-r)(x-s)$
y-int is "c"	vertex is (h,k)	x-int are "r" and "s"

"a" in all three forms is the same and indicates the direction of opening and the vertical stretch/compression

Completing the Square

Why do we need this process?

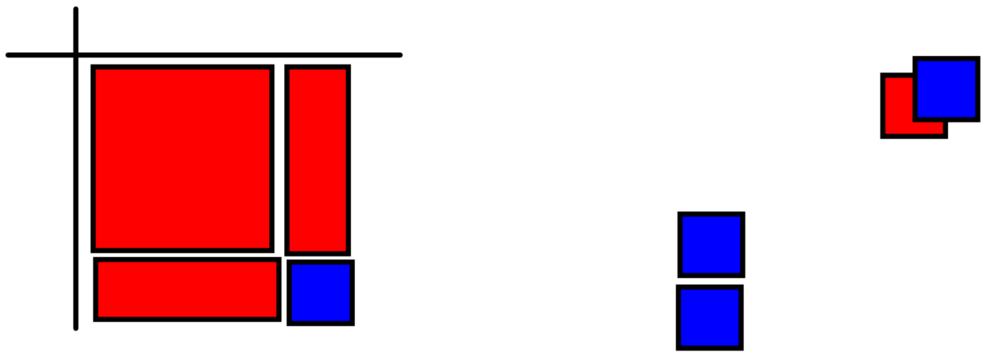
This is the operation from std $\Rightarrow$ vertex form

What it is: $f(x)=ax^2+bx+c \longrightarrow f(x)=a(x-h)^2+k$

What it is not: FACTORING or SOLVING

RECALL: Completing the Square with Algetiles $f(x) = a(x-h)^2 + k$

$x^2 + 2x - 3$



$$\therefore \quad x^2 + 2x - 3 = (x+1)^2 - 4$$

You are forcing part of the trinomial to be a perfect square which can then be factored to a binomial squared.

$f(x) = ax^2 + bx + c$ $\xrightarrow{\text{complete the square}}$ $f(x) = a(x-h)^2 + k$

$\xleftarrow{\text{expand and simplify}}$

- hard to graph
- vertex unknown

- easy to graph
- vertex & transformations known

What information can you read from vertex form? $f(x) = a(x-h)^2 + k$

- direction of opening and stretch factor 'a'
- the vertex (h,k)
- max/min value & when it occurs

Ex. 1 Find the value of c that makes a perfect square trinomial.

a) $x^2 + 8x + c$

$\frac{8}{2} = 4$ 4^2

$c = 16$

b) $x^2 - 10x + c$

-5

$(-5)^2$

$c = 25$

c) $x^2 + 3x + c$

$\frac{3}{2}$ $\left(\frac{3}{2}\right)^2$

$= \frac{9}{4}$

Ex. 2 Change to vertex form by completing the square.

a) $f(x) = x^2 - 12x + 8$

$= \underbrace{x^2 - 12x + 36}\ -36 + 8$

$= (x-6)^2 - 36 + 8$

$= (x-6)^2 - 28$

Process

⇒ Common factor the first two terms so the coefficient of x^2 is 1.

⇒ Add and subtract the number that will make a perfect square trinomial.

⇒ Remove the compensation term from the brackets.

⇒ Factor the perfect square and collect the remaining terms.

b) $f(x) = 2x^2 - 12x + 23$

$= 2(x^2 - 6x) + 23$

$= 2(x^2 - 6x + 9 - 9) + 23$

$= 2(x^2 - 6x + 9) - 18 + 23$

$= 2(x-3)^2 + 5$

Ex. 3 Determine the $\underbrace{\text{max/min value and when it occurs}}_{\text{Vertex!}}$.

a) $f(x) = -4x^2 - 5x - 3$

$= -4\left(x^2 + \frac{5}{4}x + \frac{25}{64} - \frac{25}{64}\right) - 3$

$= -4\left(x^2 + \frac{5}{4}x + \frac{25}{64}\right) + \frac{25}{16} - 3$

$= -4\left(x + \frac{5}{8}\right)^2 + \frac{25}{16} - \frac{48}{16}$

$= -4\left(x + \frac{5}{8}\right)^2 - \frac{23}{16}$

$\therefore$ Max value of $-\frac{23}{16}$

Occurs when x is $-\frac{5}{8}$

b) $f(x) = \frac{2}{3}x^2 + 7x - \frac{1}{2}$

$= \frac{2}{3}\left(x^2 + \frac{21}{2}x + \frac{441}{16} - \frac{441}{16}\right) - \frac{1}{2}$

$7 \div \frac{2}{3} = 7 \times \frac{3}{2}$

$= \frac{2}{3}\left(x^2 + \frac{21}{2}x + \frac{441}{16}\right) - \frac{147}{8} - \frac{1}{2}$

$-\frac{\cancel{441}^{147}}{\cancel{16}_{8}} \times \frac{\cancel{2}}{\cancel{3}} = -\frac{147}{8}$

$= \frac{2}{3}\left(x + \frac{21}{4}\right)^2 - \frac{151}{8}$

$\therefore$ Min value of $-\frac{151}{8}$

Occurs when $x = -\frac{21}{4}$

Ex. 4 A football player kicks a ball off a football tee. The height of the ball, h, in metres after t seconds, can be modelled using the formula: $h(t) = -5t^2 + 20t$. What is the $\underbrace{\text{maximum height}}_{\text{vertex}}$ of the ball?

vertex y value

$$h(t) = -5(t^2 - 4t + 4 - 4)$$
$$= -5(t^2 - 4t + 4) + 20$$
$$= -5(t - 2)^2 + 20$$

$\therefore$ Max height of the ball is 20m

Ex. 5 Rachel is selling scarves at a craft show. Each scarf costs $6 to make. She wants to sell, at $10 per scarf, the same amount as last year, when they sold 40 scarves. For every 50 cent increase in the price, she expects to sell four fewer scarves. What selling price will **maximize their profit** and what will the profit be?

Understand the Problem

What was Rachel's profit last year?

Profit = (Selling Price - Cost)(Quantity Sold)
= ( )( )
= ( )( )
= $

Profit $_{\text{wrt increase in price}}$ = (Profit Price)(# Sold)

Let x represent # of $0.50 increases

"Profit price" is $4 $+0.5x$

"# sold" is $40-4x$

$$P=(4+0.5x)(40-4x)$$
$$=160-16x+20x-2x^2$$
$$=-2x^2+4x+160$$

Need Max!

$$P=-2(x^2-2x+1-1)+160$$
$$=-2(x^2-2x+1)+2+160$$
$$=-2(x-1)^2+162$$

Vertex $(1, 162)$

# of price increases!

Profit

Price $= \$10+0.50(1)$
$= \$10.50$

∴ She should increase her price 1 time
New selling price is 10.50
Max profit is $162

Graph

Homework
p. 31 # 1ace, 2bcef, 5-9, 11

I just completed this.
I think math ~~guys~~ people will love it.