1.3 Factoring

take out brackets

$$3(x+2) \underset{\text{Factor}}{\overset{\text{Expand/Multiply}}{\rightleftarrows}} 3x+6$$

To factor is to write an algebraic expression as a product of two or more other algebraic expressions .

Why factor? To arrive at equivalent expressions which are presented in simpler terms which allows us to:
- Solve equations
- Graph relations

In grade 10 you learned how to:
- Common Factor
- Factor by Grouping
- Factor Simple Trinomials
- Factor Complex Trinomials
- Factor a Difference of Squares
- Factor a Perfect Square Trinomial

Common Factoring

Always your first and last step.

2 or more terms

- Take out the greatest common factor.
- Divide the expression by the GCF to find the other factor.

a) $2mn-4mnt$

$=2mn(1-2t)$

b) $6t^5-9t^2$

$=3t^2(2t^3-3)$

c) $3x^4-6x^3+9x$

$=3x(x^3-2x^2+3)$

d) $4x(a-b)-3(a-b)$

$=(a-b)(4x-3)$

Factor by Grouping

WHEN?

An even # of terms: 4, 6, 8, etc...

HOW?

- Group terms to form pairs.
- Factor the pairs by finding common factors.
- Factor out the shared common binomial factor.

a) $3x(m-5)+2(5-m)$

$= 3x(m-5)+2(-1)(-5+m)$

$= 3x(m-5)-2(m-5)$

$= (m-5)(3x-2)$

!

The terms *m - 5* and *5 - m* are opposites. This means that one divided by the other is *-1*.

b) $x(y-2)-4(2-y)$

$= x(y-2)+4(y-2)$

$= (y-2)(x+4)$

c) $mx+2y+my+2x$

$= mx+my+2x+2y$

$= m(x+y)+2(x+y)$

$= (x+y)(m+2)$

d) $22vx-6vy+11wx-3wy$

$= 22vx+11wx-6vy-3wy$

$= 11x(2v+w)-3y(2v+w)$

$= (2v+w)(11x-3y)$

e) y^2+1-y^3-y

$= 1(y^2+1)-y(y^2+1)$

$= (y^2+1)(1-y)$

f) $\underbrace{16x^5+8x^4}\ \underbrace{-6x^3-3x^2}\ \underbrace{+4x+2}$

$= 8x^4(2x+1)-3x^2(2x+1)+2(2x+1)$

$= (2x+1)(8x^4-3x^2+2)$

Simple Trinomials

3 terms

$ax^2 + bx + c$ where $a = 1$

$(x + n_1)(x + n_2)$

M = ac
A = b
N = n_1, n_2

a) $x^2 - 9x + 14$

M 14
A −9
N −2, −7

$= (x-2)(x-7)$

b) $5x^2 + 15x - 140$

$= 5(x^2 + 3x - 28)$

$= 5(x+7)(x-4)$

M −28
A 3
N 7, −4

c) $a^2 + 8ab + 15b^2$

M 15
A 8
N 3, 5

$= (a+3b)(a+5b)$

$a^2 + 3ab + 5ab + 15b^2$

$= a(a+3b) + 5b(a+3b)$

$= (a+3b)(a+5b)$

☆ NOT NECESSARY ☆

d) $x^4 + 2x^2b - 24b^2$

$= (x^2 - 4b)(x^2 + 6b)$

M −24
A 2
N −4, 6

Difference of Squares

2 terms

2 perfect squares separated by a subtraction: $a^2 - b^2$

$a^2 - b^2 = (a - b)(a + b)$

conjugates

a) $49x^2 - 16y^2$

$= (7x + 4y)(7x - 4y)$

b) $3x^2 - 12$

$= 3(x^2 - 4)$

$= 3(x + 2)(x - 2)$

$(a)^2$ $\left(\frac{1}{3}\right)^2$

c) $a^2 - \frac{1}{9}$

$= \left(a + \frac{1}{3}\right)\left(a - \frac{1}{3}\right)$

d) $81 - m^{12}$

$= (9 + m^6)(9 - m^6)$

$= (9 + m^6)(3 + m^3)(3 - m^3)$

e) $(3x - 2)^2 - (5x + 1)^2$

$= [(3x - 2) - (5x + 1)][(3x - 2) + (5x + 1)]$

$= (3x - 2 - 5x - 1)(3x - 2 + 5x + 1)$

$= (-2x - 3)(8x - 1)$

Complex Trinomials

3 terms

$ax^2 + bx + c$ where $a \neq 1$

$(a_1x + f_1)(a_2x + f_2)$

M = ac
A = b
N = n_1, n_2

1. Use a, n_1 and n_2 to find the factors.

$$\frac{a}{n_1}, \frac{a}{n_2}$$

2. Reduce.

$$\frac{a_1}{f_1}, \frac{a_2}{f_2}$$

OR

Decompose the middle term using n_1, n_2 and factor by grouping.

a) $10x^2 - 11x - 6$

$= (5x+2)(2x-3)$

M -60
A -11
N $\frac{10}{-15}, \frac{10}{4}$

$\frac{2}{-3}$ $\frac{5}{2}$

b) $14x^2 + 31xy - 10y^2$

$= (7x-2y)(2x+5y)$

M -140
A 31
N $\frac{14}{-4}, \frac{14}{35}$

$\frac{7}{-2}$ $\frac{2}{5}$

c) $18a^2b + 3ab - 6b$

$= 3b(6a^2 + a - 2)$

M -12
A 1
N $\frac{6}{4}, \frac{6}{-3}$

$\frac{3}{2}$ $\frac{2}{-1}$

$= 3b(3a+2)(2a-1)$

d) $3x^4 - 25x^2 - 18$

$= (x^2-9)(3x^2+2)$

$= (x-3)(x+3)(3x^2+2)$

M -54
A -25
N $\frac{3}{-27}, \frac{3}{2}$

$\frac{1}{-9}$

Perfect Square Trinomials

WHEN?

3 terms

$ax^2 + bx + c$

where a & c are perfect squares and
b is twice the product of their square roots.

HOW?

$$\left(\sqrt{a}x \pm \sqrt{c}\right)^2$$

same sign as b

$(m)^2$ $(5)^2$

a) $m^2 + 10m + 25$

$= (m+5)^2$

b) $2x^2 - 24x + 72$

$= 2(x^2 - 12x + 36)$

$= 2(x - 6)^2$

$(4a)^2$ 3^2

c) $16a^2 + 24a + 9$

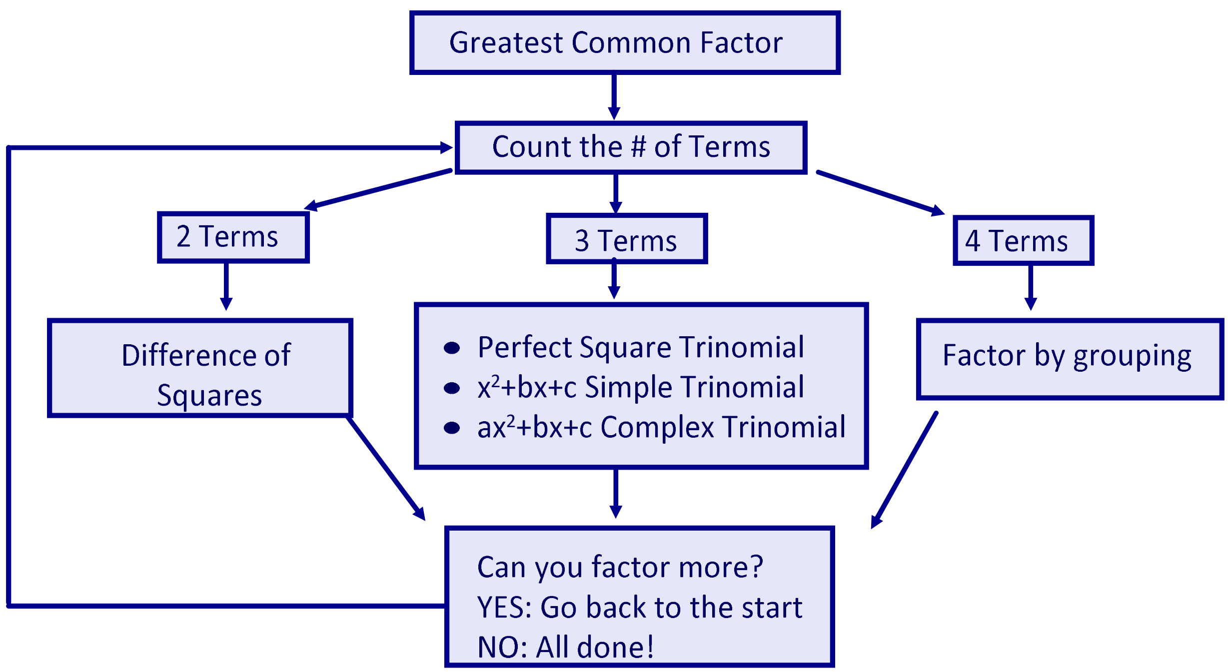
$= (4a + 3)^2$

$(x^2)^2$ $(4)^2$

d) $x^4 - 8x^2 + 16$

$= (x^2 - 4)^2$

Factoring Flowchart



Check you answer by EXPANDING

HOMEWORK
Handout 1.3