1.4B – Maximum or Minimum of a Quadratic Function MCR 3U

PART A:

1. Use partial factoring to determine the vertex of each function. State if the vertex is a minimum or a maximum.

a) $f(x) = 3x^2 - 6x + 11$
b) $f(x) = -2x^2 + 8x - 3$
c) $f(x) = \frac{1}{2}x^2 - 3x + 8$
d) $f(x) = -\frac{5}{3}x^2 + 5x - 10$
e) $f(x) = 0.3x^2 - 3x + 6$
f) $f(x) = -0.2x^2 - 2.8x - 5.4$

2. a) $f(x) = x^2 + 10x + 6$
b) $f(x) = 2x^2 + 12x + 16$
c) $f(x) = -3x^2 + 6x + 1$
d) $f(x) = -x^2 + 12x - 5$
e) $f(x) = -\frac{1}{2}x^2 - x + \frac{3}{2}$
f) $f(x) = \frac{2}{3}x^2 + \frac{16}{3}x + \frac{25}{3}$

a) (1, 8); minimum b) (2, 5); maximum
c) $\left(3, \frac{7}{2}\right)$; minimum d) $\left(\frac{3}{2}, \frac{25}{4}\right)$; maximum
e) $\left(5, -\frac{3}{2}\right)$; minimum f) $\left(-7, \frac{22}{5}\right)$; maximum

3. a) (−5, −19); minimum b) (−3, −2); minimum
c) (1, 4); maximum d) (6, 31); maximum
e) (−1, 2); maximum f) $\left(-4, -\frac{7}{3}\right)$; minimum

PART B:

1. The path of the ball for many golf shots can be modelled by a quadratic function. The path of a golf ball hit at an angle of about 10° to the horizontal can be modelled by the function $h = -0.002d^2 + 0.4d$ where h is the height of the ball, in metres, and d is the horizontal distance the ball travels, in metres, until it first hits the ground.
   a) What is the maximum height reached by the ball?
   b) What is the horizontal distance of the ball from the golfer when the ball reaches its maximum height?
   c) What distance does the ball travel horizontally until it first hits the ground?

2. The path of a basketball shot can be modelled by the equation $h = -0.09d^2 + 0.9d + 2$ where h is the height of the basketball, in metres, and d is the horizontal distance of the ball from the player, in metres.
   a) What is the maximum height reached by the ball?
   b) What is the horizontal distance of the ball from the player when it reaches its maximum height?
   c) How far from the floor is the ball when the player releases it?

3. A natural bridge is a stone arch formed over a river or stream. The longest natural bridge in the world is Rainbow Bridge in Utah. If the origin is placed at one end of the arch, the curve of the arch can be modelled by the equation $h = -0.0425d^2 + 3.57d$ where h metres represents the height and d metres represents the horizontal distance.
   a) What is the width of the arch at the base?
   b) What is the maximum height of the arch, to the nearest metre?
   c) At a horizontal distance of 10 m from the vertex, what is the height of the arch, to the nearest metre?

4. A company has found that if they increase the price of their product by two dollars, then they will sell 10 products less than before. When the initial price was $50, they sold 1000 products. The cost to make each product is $10. Determine the price that needs to be charged to attain a maximum profit and the number of products that needs to be sold to reach this maximum.

5. A grocer sells 50 loaves of bread a day at a selling price of $0.65 a loaf. The grocer estimates that for each $0.05 price increase, 2 fewer loaves of bread will be sold. What selling price will maximize the revenue?

6. Adam has 24 m of fencing to surround a garden, bounded on one side by the wall of his house. What are the dimensions of the largest rectangular garden that he can enclose?

7. The length of a rectangle is 2 m more than the width. If the area of the rectangle is 20 m^2, what are the dimensions of the rectangle, to the nearest tenth of a metre?

8. Find the two numbers whose difference is 12 and whose product is a minimum.

9. Determine an equation of each of the following quadratics (in the most appropriate form) given it:
   a) has x-intercepts of 3 and -8 and passes through the point $(4,-5)$
   b) has x-intercepts of $1+\sqrt{5}$ and $1-\sqrt{5}$ and passes through the point $(2,5)$
   c) has x-intercepts of -4 and -7 and a y-intercept of 10
   d) has an x-intercept of 9 and a minimum value of -15 when $x=-4$
   e) has a x-intercepts of $4-\sqrt{3}$ and $4+\sqrt{3}$, and a maximum value of 108

10. A soccer ball is kicked from the ground. After travelling a horizontal distance of 35 m, it just passes over a 1.5 m tall fence before hitting the ground 37 m from where it was kicked.
   a) Determine an equation of a quadratic function that can be used to model the path of the ball.
   b) Find the maximum height of the ball

11. The following function gives the height, h metres, of a batted baseball as a function of the time, t seconds, since the ball was hit. $h=-6(t-2.5)^2+38.5$
   a) What was the maximum height of the ball?
   b) What was the height of the ball when it was hit?
   c) How many seconds after it was hit did the ball hit the ground, to the nearest second?
   d) Find the height of the ball 1 s after it was hit.

12. The equation shows the height of a soccer ball, h metres, as a function of the horizontal distance, d metres, the ball travels until it first hits the ground. $h=-0.025(d-20)^2+10$
   a) What is the maximum height of the ball?
   b) What is the horizontal distance of the ball form the kicker when it reaches its maximum height?
   c) How far does the ball travel horizontally from when it was kicked until it hits the ground?
   d) What is the height of the ball when it is 10 m horizontally from the kicker?
   e) Would an opposing player positioned under the path of the ball 34 m from the kicker be able to head the ball? Explain.
   f) If the origin were placed at the vertex of the parabola, what would be the equation of the curve?

13. A touch football quarterback passed the ball to the receiver 40 m downfield. The path of the ball can be described by the function $h=-0.01(d-20)^2+6$ where h is the height of the ball, in metres, and d is the horizontal distance of the ball from the quarterback, in metres.
   a) What was the maximum height of the ball?
   b) What was the horizontal distance of the ball from the quarterback at its maximum height?
   c) What was the height of the ball when it was thrown? When it was caught?
   d) If a defensive back was 2 m in front of the receiver, how far was the defensive back from the quarterback?
   e) How high would the defensive back have needed to reach to knock down the pass?

Answers: (PART B)

1. a) 20 m b) 100 m c) 200 m 2. a) 4.25 m b) 5 m c) 2 m 3. a) 84 m b) 75 m c) 71 m
4. $130 and 600 5. $0.95 6. 12 m by 6 m 7. Width: 3.6 m; length 5.6 m 8. 6 and -6
9. a) $y=-\frac{5}{12}(x-3)(x+8)$ b) $y=-\frac{5}{4}(x^2-2x-4)$ c) $y=\frac{5}{14}(x+4)(x+7)$ d) $y=\frac{15}{169}(x+17)(x-9)$
   e) $y=-36x^2+288x-468$
10. a) $h=-\frac{3}{140}d(d-37)$ b) 7.3 m 11. a) 38.5 m b) 1 m c) 5 s d) 25 m
12. a) 10 m b) 20 m c) 40 m d) 7.5 m e) No, the ball would be at a height of 5.1 m, which is too high to jump.
   f) $h=-0.025d^2$
13. a) 6 m b) 20 m c) 2 m; 2 m d) 38 m e) 2.76 m