7.5a Modelling with Functions

All of the functions that you've studied in this course can be used to model real-life data....a function model will describe the data (not perfectly) and help to make predictions.

remember: linear, quadratic, cubic, quartic, exponential, logarithmic, rational, root, absolute value, trigonometric, inverses........and combinations!!!!!

The Logistic Function NEW!!

$$P(t)=\frac{c}{1+ab^{t}}$$

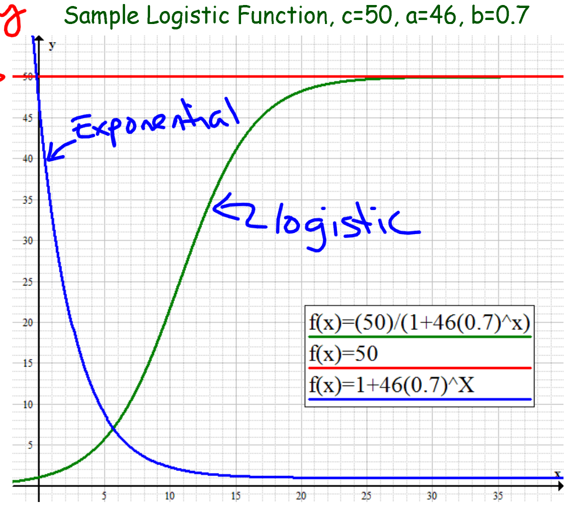
c ← linear function, c=maximum value

$1+ab^t$ ← exponential function, t= time

- a common and widely used model for growth
- it models slow growth for small values of t then rapid growth
- then slow growth again until it reaches a maximum value (max. at horiz. asympt)
- the values of a and b can be found using any 2 points on the function

Sample Logistic Function, c=50, a=46, b=0.7

maximum capacity



The quotient of a linear and exponential function.

PART A: Modelling by Hand

Ex: #1

A pond study in the back 40 has revealed that the population of a water bug that was initially 30 has grown to 240 in 5 days. If the maximum capacity of the pond is 1000 of these bugs, how long will it take to reach the maximum?

Let x be time in days
y be # of bugs

a) use a linear model (y=mx+b)

$m = \frac{y_2 - y_1}{x_2 - x_1}$ b = initial $= 30$

$= \frac{240-30}{5-0}$

$= 42$

$\therefore y = 42x + 30$

1000 ?

$1000 = 42x + 30$

$x \doteq 23$ days

b) use an exponential model ($y = ca^x$)

(0,30)
(5,240)

c = initial $= 30$

$y = ca^x$

$240 = 30a^5$

$8 = a^5$

$\sqrt[5]{8} = a$

$a \doteq 1.52$

$\therefore y = 30(1.52)^x$

1000 ?

$\frac{1000}{30} = (1.52)^x$

$\log\left(\frac{1000}{30}\right) = x \cdot \log(1.52)$

$x = \frac{\log\left(\frac{1000}{30}\right)}{\log(1.52)}$

$\doteq 8.4$ days

c) use a logistic model

$P(t) = \frac{c}{1+ab^t}$

c = maximum $= 1000$

$y = \frac{1000}{1+ab^x}$

(0,30)
(5,240)

① Use (0,30) to solve for a

$30 = \frac{1000}{1+ab^0}$

$30(1+a) = 1000$

$30 + 30a = 1000$

$a \doteq 32.3$ (Store this!)

② Use (5,240) & a = 32.3 to solve for b

$240 = \frac{1000}{1+(32.3)b^5}$

$240 + (7759.99)b^5 = 1000$

$\frac{\cancel{7759.99}\, b^5}{\cancel{7759.99}} = \frac{760}{7759.99}$

$b^5 = 0.098$

$b \doteq 0.63$

(Store this!)

③ 1000 ?

$a = 32.3$ & $b = 0.63$

$\therefore y = \frac{1000}{1+(32.3)(0.63)^x}$

$999.99 = \frac{1000}{1+(32.3)(0.63)^x}$

1000 is HA
Use 999.99 Instead

$1 + (32.3)(0.63)^x = \frac{1000}{999.99}$

$(32.3)(0.63)^x = \frac{1000}{999.99} - 1$

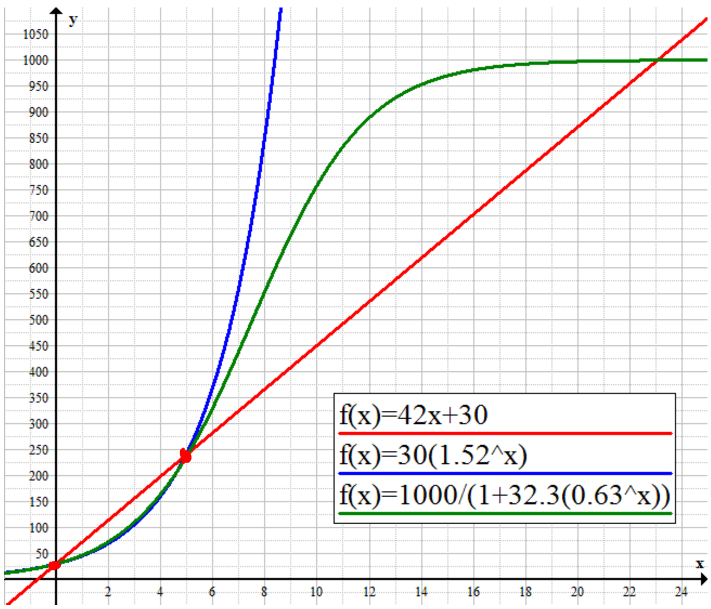
$(0.63)^x = \left(\frac{1000}{999.99} - 1\right) \div 32.3$

$x \cdot \log(0.63) = \log\left[\left(\frac{1000}{999.99} - 1\right) \div 32.3\right]$

$x = \frac{\log\left[\left(\frac{1000}{999.99} - 1\right) \div 32.3\right]}{\log(0.63)}$

$\doteq 32.4$ days

Graphs from Ex. #1:



Discuss: Which is a better model????

Jan 10-9:15 PM

Modelling Using Graphing Technology

Ex: 2 The table below shows the amount of water vapour (mL of water/m^3 of air) in the air as a function of temperature (°C).

Temp	Vapour
0	4.8
5	6.8
10	9.4
15	12.8
20	17.3
25	23.1
30	30.4
35	39.6

a) Create a scatter plot of the data.

b) Use regression to find the value of R^2 for each model:

linear: $R^2 = 0.9422$

quadratic: $R^2 = 0.999$

cubic: $R^2 = 1$

exponential: $R^2 = 0.9982$

other???

Are not 'ideal' models

R^2: this value is the fraction of variance in y that is explained by the model based on x. In general, values closer to 1.0 (ie. 100%) are better fits......though this is NOT always the case. The model must also MAKE SENSE with the given data.

Homework
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#1,2,4,5,12... pencil & paper

#7,8,11, **need graphing technology