5.1 Exploring Logarithmic Functions

The inverse of the exponential function $y=a^x$ can be written:

- ☆ in exponential form as $x=a^y$
- ☆ in logarithmic form as $y=\log_a x$

y is the exponent that must be applied to base "a" to get x

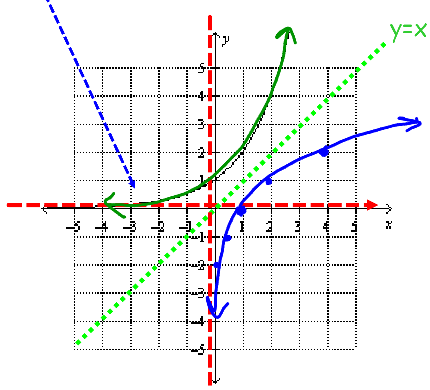
Consider the following exponential functions and their inverses:

Inverse

A) $y=2^x$ ⟶ $x=2^y$ or $\log_2 x=y$

$y=2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4



$y=\log_2 x$

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

Reminder: Exponential Function $y=a^x$ where $a>0$ and $a\neq 0$

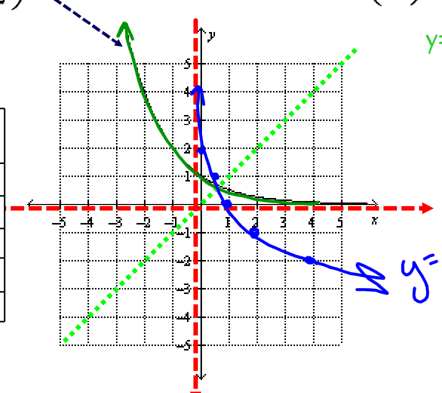
- ☺ H.A. ⇒ $y=0$
- ☺ y-intercept is 1
- ☺ Function is increasing if $a>1$ Growth
- ☺ Function is decreasing if $0<a<1$ Decay

Inverse

B) $y=\left(\frac{1}{2}\right)^x$ ⟶ $x=\left(\frac{1}{2}\right)^y$ or $y=\log_{\frac{1}{2}} x$

$y=\left(\frac{1}{2}\right)^x$

x	y
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$



$y=\log_{\frac{1}{2}} x$

$y=\log_{\frac{1}{2}} x$

x	y
4	-2
2	-1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2

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Gizmos Demo

Note the similarities and differences between the exponential and the logarithmic functions:

	Exponential $y=a^x$	Logarithmic $y=\log_a x$
$a>1$	**increasing**	**increasing** as $x \to \infty$, $y \to \infty$
$0<a<1$	**decreasing**	**decreasing**
asymptote	**HA @ y = 0**	**VA @ x = 0**
intercept	**y-int @ 1**	**x-int @ 1**
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}, x > 0$
Range	$y \in \mathbb{R}, y > 0$	$y \in \mathbb{R}$

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Ex 1 For each of the following:

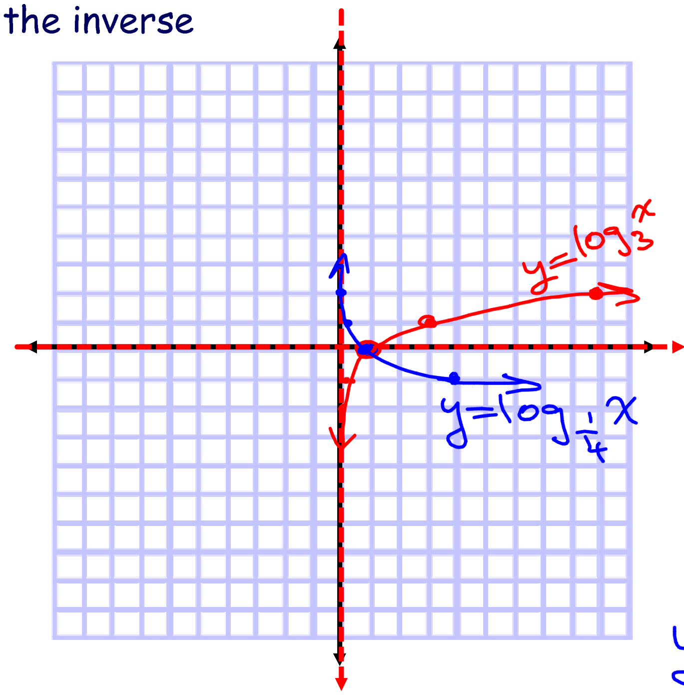
a) determine the equation of the inverse

b) graph the inverse

i) $f(x)=3^x$

x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

$y=\log_3 x$



ii) $f(x)=\left(\frac{1}{4}\right)^x$

x	y
-2	16
-1	4
0	1
1	$\frac{1}{4}$
2	$\frac{1}{16}$

$y=\log_{\frac{1}{4}} x$

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Ex 2 Convert the following to exponential or logarithmic form.

$y = a^x \Leftrightarrow x = \log_a y$

$9 = 3^2 \Leftrightarrow 2 = \log_3 9$

a) $y=\log_7 x$

$7^y = x$

b) $4^x=y$

$x = \log_4 y$

c) $\log_3 81=4$

$3^4 = 81$

Ex 3 Evaluate the following.

a) $\log_3 81$

$3^x = 81$

$3^x = 3^4$

$\therefore x = 4$

OR

$\log_3 3^4$

$= 4$

b) $\log_5\sqrt{25}$

$5^x = \sqrt{25}$

$5^x = 5^1$

$\therefore x = 1$

OR

$\log_5 5$

$= 1$

c) $\log_4 x=2$

$4^2 = x$

$16 = x$

d) $x=\log_4(\frac{1}{16})$

$4^x = \frac{1}{16}$

$4^x = 4^{-2}$

$\therefore x = -2$

Homework 5.1:
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