3.4 Stretches of Periodic Functions

When sketching sine and cosine functions, remember the 5 key points: Maximum, Minimum, and zeroes. These 5 points are equally spaced along the x-axis, so they divide the period into quarters.

- to find the scale: period divided by 4

A) Vertical Stretches or Compressions

Given $y = af(x)$

- is called the amplitude

When,

$|a| > 1$ Vertical stretch by factor of a

Looks like compression " " " " " "

$0 < |a| < 1$

$a < 0$ (*i.e. a is negative*) Reflection about x-axis

Therefore, y=asinx and y=acosx results in Vertical Stretch

where "a"is called the amplitude.

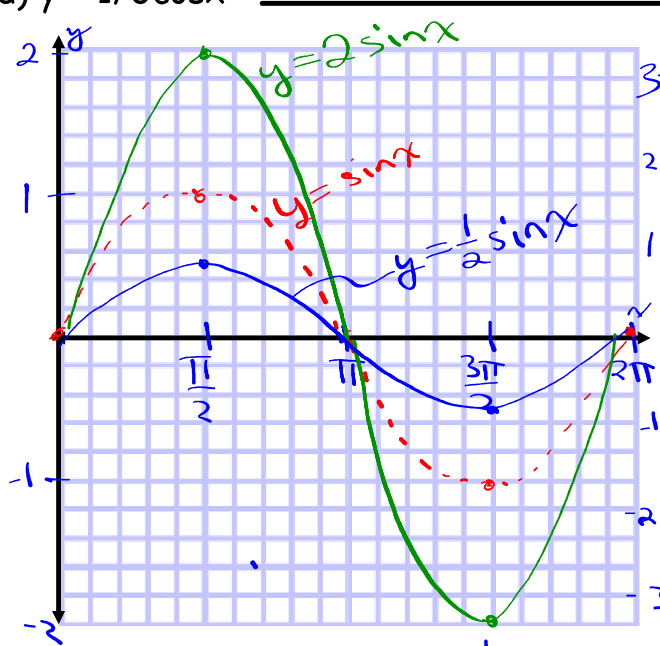
Ex 1: Sketch and describe the transformations with respect to y=sinx for a,b and y = cosx for c,d for one cycle

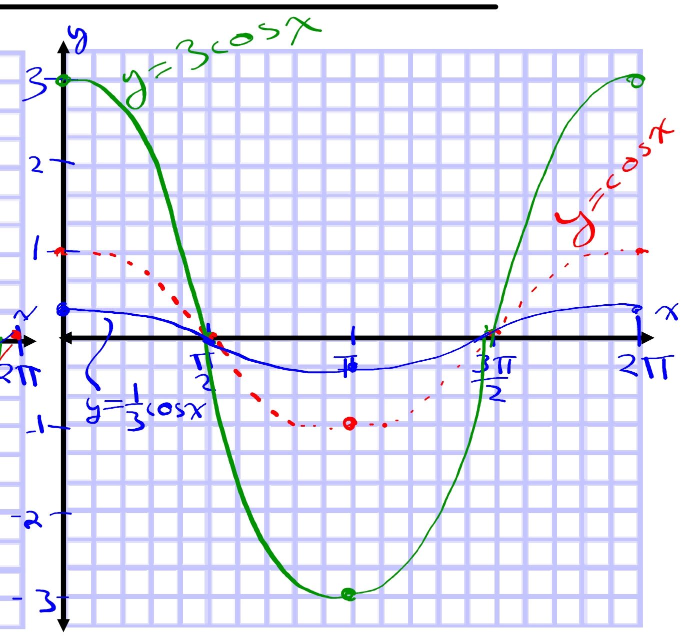
a) y = 2sinx Vertical stretch by a factor of 2

b) y = 1/2sinx " " " $\frac{1}{2}$

c) y = 3cosx " " " 3

d) y = 1/3cosx " " " $\frac{1}{3}$





Does the amplitude change? Yes

Does the period change? No

B) Horizontal Stretches or Compressions

Given y = f(kx)

Remember, k is INSIDE the function and behaves OPPOSITE from what you would expect.

When,

$|k| > 1$ horizontal stretch by factor $\frac{1}{K}$

Looks like a stretch

$0 < |k| < 1$ " " " $\frac{1}{K}$

$k < 0$ (*i.e. k is negative*) Reflection about y-axis

Because you are stretching/compressing horizontally, the period would change.

$$\boxed{period = \frac{2\pi}{k}}$$

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Ex 2: Sketch and describe transformations with respect to y=cosx for a and b and y=sinx for c and d for one cycle.

a) $y=\cos 2x$ horizontal stretch by factor of $\frac{1}{2}$

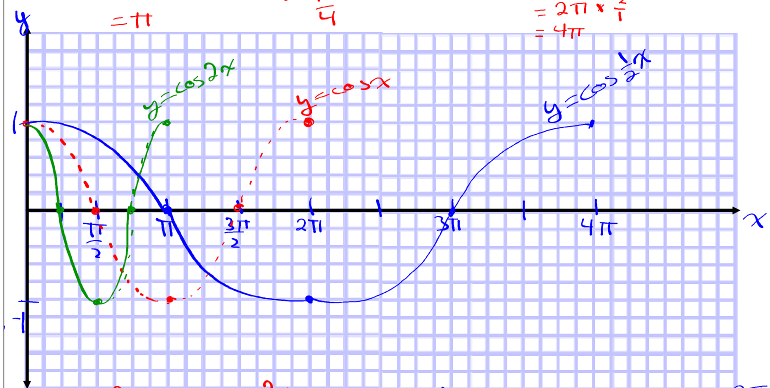
b) $y=\cos \frac{1}{2}x$ " " " 2

c) $y=\sin 3x$ " " " $\frac{1}{3}$

d) $y=\sin \frac{1}{3}x$ " " " 3

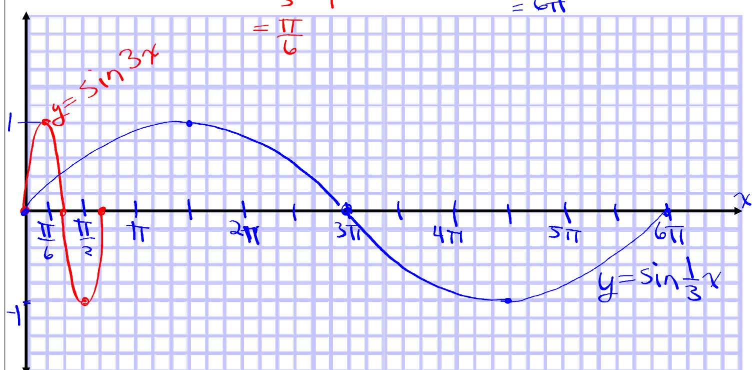
a) Per $= \frac{2\pi}{2} = \pi$ Scale $=$ per $\div 4 = \frac{\pi}{4}$

b) Per $= \frac{2\pi}{\frac{1}{2}} = 2\pi \times \frac{2}{1} = 4\pi$ Scale $= \pi$



c) Per $= \frac{2\pi}{3}$ Scale $= \frac{2\pi}{3} \div 4 = \frac{2\pi}{3} \times \frac{1}{4} = \frac{\pi}{6}$

d) Per $= \frac{2\pi}{\frac{1}{3}} = 6\pi$ Scale $= \frac{3\pi}{2}$



Did the amplitude change? NO

Did the period change? YES!

$$\text{Per} = \frac{2\pi}{K} \qquad |a| = \frac{\max - \min}{2}$$

$$K = \frac{2\pi}{\text{Per}}$$

Question #7

Homework:

Handout 3.4