

1.  $\sin(a+b) + \sin(a-b) = 2\sin a \cos b$
2.  $\sin x \tan x = \sec x - \cos x$
3.  $\sin(p+q)\sin(p-q) = \sin^2 p - \sin^2 q$
4.  $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$
5.  $\frac{\cos(x-y)}{\cos x \sin y} = \tan x + \cot y$
6.  $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$
7.  $2\cos x \cos y = \cos(x+y) + \cos(x-y)$
8.  $\cos^2 x \cos^2 y + \sin^2 x \sin^2 y + \sin^2 x \cos^2 y + \sin^2 y \cos^2 x = 1$
9.  $2\cos x \cos y = \cos(x+y) + \cos(x-y)$
10.  $\sec^2 x - \sec^2 y = \tan^2 x - \tan^2 y$  ?
11.  $\sin 4x = 4\cos x \sin x (2\cos^2 x - 1)$
12.  $\frac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$
13.  $\cot\left(\frac{5\pi}{4} + x\right) = -\frac{\tan x - 1}{\tan x + 1}$  ?
14.  $(\sec x - \cos x)(\csc x - \sin x) = \frac{\tan x}{1 + \tan^2 x}$
15.  $\frac{\cos(x-y) - \cos(x+y)}{\cos(x-y) + \cos(x+y)} = \tan x \tan y$
16.  $\cos^6 x + \sin^6 x = 1 - 3\sin^2 x + 3\sin^4 x$
17.  $\cot a + \tan a = 2\csc 2a$
18.  $\sec^6 x - \tan^6 x = 1 + 3\tan^2 x \sec^2 x$
19.  $\frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = \frac{2 + \sin 2x}{2}$
20.  $\frac{\sin x + \sin 2x + \sin 3x}{\cos x + \cos 2x + \cos 3x} = \tan 2x$  ?
21.  $\sin^2 x = \frac{1 - \cos 2x}{2}$

# 7.4 Trig Identities

$$= \sin(a+b) + \sin(a-b)$$

$$= \sin a \cos b + \sin b \cos a + \sin a \cos b - \cos a \sin b$$

$$= 2 \sin a \cos b \quad \checkmark$$

4) L.S.

$$= \cos^4 x - \sin^4 x$$

$$= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$$

$$= 1 - \sin^2 x - \sin^2 x$$

$$= 1 - 2 \sin^2 x \quad \checkmark$$

5) L.S.

$$= \frac{\cos(x-y)}{\cos x \sin y}$$

$$= \frac{\cos x \cos y + \sin x \sin y}{\cos x \sin y}$$

$$= \frac{\cos x \cos y}{\cos x \sin y} + \frac{\sin x \sin y}{\cos x \sin y}$$

$$= \cot y + \tan x \quad \checkmark$$

6) L.S.

$$= \frac{1}{\sin^2(x)} + \frac{1}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$$

$$= \frac{1}{\sin^2 x \cos^2 x}$$

$$= \csc^2 x \sec^2 x \quad \checkmark$$

7) R.S.

$$= \cos x \cos y + \sin x \sin y + \cos x \sin y - \sin x \cos y$$

$$= 2 \cos x \cos y \quad \checkmark$$

8) L.S.

$$= \cos^2 x \cos^2 y + \sin^2 x \sin^2 y + \sin^2 x \cos^2 y + \sin^2 y \cos^2 x$$

$$= \cos^2 y (\cos^2 x + \sin^2 x) + \sin^2 y (\sin^2 x + \cos^2 x)$$

$$= 1 \quad \checkmark$$

9) R.S.

$$= \cos x \cos y + \sin y \sin x + \cos x \cos y - \sin y \sin x$$

$$= 2 \cos x \cos y$$

10) L.S.

$$= \frac{1}{\cos^2 x} - \frac{1}{\cos^2 y}$$

$$= \frac{\cos^2 y - \cos^2 x}{\cos^2 x \cos^2 y}$$

$$= \frac{(1 - \sin^2 y) - (1 - \sin^2 x)}{\cos^2 x \cos^2 y}$$

$$= \frac{-\sin^2 y + \sin^2 x}{\cos^2 x \cos^2 y} \quad \checkmark$$

R.S.

$$= \frac{\sin^2 x + \sin^2 y}{\cos^2 x \cos^2 y}$$

11) L.S.

$$= \sin 2(\theta x)$$

$$= 2 \sin 2x \cos 2x$$

$$= 2(2 \sin x \cos x)(2 \cos^2 x - 1)$$

$$= 4 \sin x \cos x (2 \cos^2 x - 1) \quad \checkmark$$

14) L.S.

$$\left( \frac{1}{\cos x} - \cos x \right) \left( \frac{1}{\sin x} - \sin x \right)$$

$$= \left( \frac{1 - \cos^2 x}{\cos x} \right) \left( \frac{1 - \sin^2 x}{\sin x} \right)$$

$$= \frac{\sin^2 x \cdot \cos^2 x}{\cos x \sin x} \quad \checkmark = \sin x \cos x$$

12) L.S.

$$\frac{\frac{1}{\tan x} + \frac{1}{\tan y}}{\frac{1}{\tan x} + \frac{1}{\tan y}}$$

$$= \frac{\tan x + \tan y}{\tan y + \tan x}$$

$$= \frac{\tan x \cdot \tan y}{\tan y + \tan x} \quad \checkmark$$

13)  $\cot\left(\frac{5\pi}{4}\right) + \cot x$

$$= \frac{1 + \cot x}{1 - \cot x}$$

$$= \frac{\tan + 1}{\tan x}$$

$$= \frac{\tan - 1}{\tan x}$$

$$= \frac{\tan + 1}{\tan - 1}$$

R.S.

$$\frac{\tan x}{1 + \tan^2 x}$$

$$= \frac{\sin x}{\cos x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{\cos x \sin x}{1}$$

$$\begin{aligned}
 15) \quad & \text{L.S.} \\
 & \cos x \cos y + \sin x \sin y - \cos x \cos y + \sin x \sin y \\
 & \cos x \cos y + \sin x \sin y + \cos x \cos y - \sin x \sin y \\
 & = \cos y (\cos x - \cos x) + \sin y (2 \sin x) \\
 & \quad \cos y (\cos x + \cos x) + \sin y (-\sin x - \sin x) \\
 & = \tan y \tan x \checkmark
 \end{aligned}$$

$$\begin{aligned}
 16) \quad & \text{L.S.} \\
 & (1 - \sin^2 x)(1 - \sin^2 x)(1 - \sin^2 x) + \sin^6 x \\
 & = (1 - 2\sin^2 x + \sin^4 x)(1 - \sin^2 x) + \sin^6 x \\
 & = 1 - \sin^2 x - 2\sin^2 x + 2\sin^4 x + \sin^4 x - \sin^6 x + \sin^6 x \\
 & = 1 - 3\sin^2 x + 3\sin^4 x \checkmark
 \end{aligned}$$

$$\begin{aligned}
 17) \quad & \text{L.S.} \\
 & = 2 \csc 2a \\
 & = \frac{2}{\cos 2a \sin a} \\
 & = \frac{1}{\cos a \sin a} \\
 & \text{R.S.} \\
 & = \frac{\cos a + \sin a}{\sin a \cos a} \\
 & = \frac{\cos^2 a + \sin^2 a}{\sin a \cos a} \\
 & = \frac{1}{\cos a \sin a} \checkmark
 \end{aligned}$$

$$18) \quad \sec^6 x - \tan^6 x$$

$$= \frac{1}{\cos^6 x} - \frac{\sin^6 x}{\cos^6 x}$$

$$= \frac{1 - \sin^6 x}{\cos^6 x}$$

$$= \frac{1 - (1 - \cos^2 x)(1 + \cos^2 x) \sin^2 x}{\cos^6 x} = 1 + \sin x \cos x$$

$$= \frac{1 - (1 - 2\cos^2 x + \cos^4 x) \sin^2 x}{\cos^6 x}$$

$$= \frac{1 - \sin^2 x + 2\cos^2 x \sin^2 x - \cos^4 x \sin^2 x}{\cos^6 x}$$

$$= \frac{\cos^2 x + 2\cos^2 x(1 - \cos^2 x) - \cos^4 x(1 - \cos^2 x)}{\cos^6 x}$$

$$= \frac{\cos^2 x + 2\cos^2 x - 2\cos^4 x - \cos^4 x + \cos^6 x}{\cos^6 x}$$

$$= \frac{3\cos^2 x - 3\cos^4 x + \cos^6 x}{\cos^6 x}$$

$$= \frac{\cos^6 x}{\cos^6 x} + \frac{3\cos^2 x(1 - \cos^2 x)}{\cos^6 x}$$

$$= 1 + \frac{3\sin^2 x}{\cos^4 x}$$

$$= 1 + 3 \tan x \sec^2 x \checkmark$$

$$21) \quad \text{R.S.}$$

$$= \frac{1 - (1 - 2\sin^2 \theta)}{2}$$

$$= \sin^2 \theta \checkmark$$

$$19) \quad \text{L.S.}$$

$$= \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x}$$

$$= \frac{(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)}{\sin x - \cos x}$$

$$\#20) \quad \text{R.S.} \\
 = \tan 2x$$

$$= \frac{2 \sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$\text{L.S.} \\
 = \frac{\sin x + 2 \sin x \cos^2 x + \sin^3 x \cos x + \cos^2 x \sin x}{\cos x + (1 - 2\sin^2 x) + \cos^2 x \cos x - \sin^2 x \sin x}$$

$$= \frac{\sin x + 2 \sin x \cos^2 x + 2 \cos^2 x \sin x - (1 - 2\sin^2 x) \sin x}{\cos x + (1 - 2\sin^2 x) + (1 - 2\sin^2 x) \cos x - 2 \cos x \sin^2 x}$$

$$= \frac{2 \sin x \cos^2 x + 2 \cos^2 x \sin x + 2 \sin^3 x}{2 \cos x - 2 \sin^2 x + 1 - 2 \sin^2 \cos x - 2 \cos x \sin^2 x}$$