### 3.2 Increasing/Decreasing Functions

Increasing
the graph rises from
left to right
on an interval for any value
of $x_{1}<x_{2}, f\left(x_{1}\right)<f\left(x_{2}\right)$
slope of the tangent is positive

$$
f^{\prime}(x)>0 \quad f^{\prime}(x)<0
$$

Ex. 1 Determine the intervals of increase and decrease, and local extrema for the following functions.

$$
\text { a) } \begin{aligned}
& f(x)=x^{4}-4 x^{3}+4 x^{2} \\
& f^{\prime}(x)=4 x^{3}-12 x^{2}+8 x \\
& 0=4 x\left(x^{2}-3 x+2\right) \\
& 0=4 x(x-2)(x-1) \\
& x=0,2,1
\end{aligned}
$$

| - | + | - | + |
| :---: | :---: | :---: | :---: |
| $\underset{\text { MIN }}{0} \underset{\text { MAX }}{1}$ | $\underset{\text { MIN }}{2}$ |  |  |

Set $f^{\prime}(x)=0$
Solve to find critical \#s
$\therefore$ Increasing from $(0,1)$

Decreasing from $(-\infty, 0)$ \& $(1,2)$

$$
\begin{aligned}
& f(0)=0 \\
& f(1)=1 \\
& f(2)=0
\end{aligned}
$$

b) $g(x)=\frac{4 x}{x^{2}+2}$

$$
\begin{aligned}
g^{\prime}(x) & =\frac{4\left(x^{2}+2\right)-4 x(2 x)}{\left(x^{2}+2\right)^{2}} \\
& =\frac{4 x^{2}+8-8 x^{2}}{\left(x^{2}+2\right)^{2}} \\
& =\frac{-4 x^{2}+8}{\left(x^{2}+2\right)^{2}} \\
0 & =-4 x^{2}+8 \\
8 & =4 x^{2} \\
x & = \pm \sqrt{2}
\end{aligned}
$$


$\therefore$ Increasing from $(-\sqrt{2}, \sqrt{2})$
Decreasing from
$4(-\infty,-\sqrt{2})$
$(\sqrt{2}, \infty)$

$$
\begin{aligned}
& f(-\sqrt{2})=-\sqrt{2} \\
& f(\sqrt{2})=\sqrt{2}
\end{aligned}
$$

Ex. 2 Given the graph of $f(x)$, find the intervals where $f(x)$ is increase and decreasing. Prepare a strip for $f(x)$ and $f^{\prime}(x)$.
a)

$\begin{array}{cc}\text { Asymptote } \\ & \\ & \\ & \text { Zero }\end{array}$

b)


Zero Asymptote


Increasing ( $-4,2$ )
Decreasing $(-\infty,-4)+(2, \infty)$

d)


Ex. 3 Sketch a graph that has the following properties.
$f^{\prime}(x)>0$ for $x<1.4$ and $x>5.5$
$f^{\prime}(x)<0$ for $1.4<x<5.5$
$f^{\prime}(1.4)=f^{\prime}(5.5)=0$
$f(1.4)=6.4$ and $f(5.5)=-10.4$



## Homework page 170 \#3, 5,7-11



